

Mathematics Unit: *DVD Sales*

From the PISA 2022 Field Trial

The DVD Sales Scenario

PISA 2022


DVD Sales
Introduction

Read the introduction. Then click on the NEXT arrow.

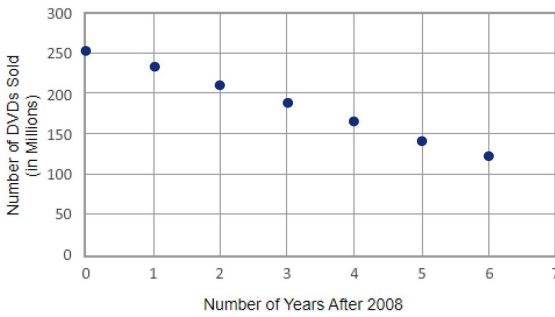
DVD SALES

The total number of DVDs sold each year in the United Kingdom from 2008 to 2014 is shown in the graph below.

The values on the horizontal axis represent the number of years **after** 2008. Place the cursor over the points in the graph to see the coordinates of that point. For example, the point (0, 252.9) indicates that there were 252.9 million DVDs sold in 2008. The point (1, 234.6) indicates that there were 234.6 million DVDs sold during the year 2009, etc.



DVD Sales in the United Kingdom



Number of Years After 2008	Number of DVDs Sold (in Millions)
0	252.9
1	234.6
2	213.4
3	191.2
4	168.9
5	146.7
6	124.5

This is the introduction screen for the unit *DVD Sales*. Aside from introducing the scenario, this introduction provides students with information on how to read and interpret the data shown in the graph and gives them a chance to practice placing the cursor over the points to see the specific data that appear, which will be necessary for responding to the items. For the first two items in this unit, the data are presented in this format where the independent variable is the number of years after 2008 and the dependent variable is number of DVDs sold, in millions. For example, the point (0, 252.9) corresponds to the year 2008 with 252.9 million DVDs sold that year; the point (1, 234.6) corresponds to the year 2009 (i.e., 1 year after 2008) with 234.6 million DVDs sold that year; etc.

The DVD Sales Released Item #1

PISA 2022

DVD Sales
Question 1 / 3

Refer to "DVD Sales" on the right. Click on the choices in the table to answer the question.

Are the statements in the table below supported by the information shown in the graph? Click on either **Yes** or **No** for each statement.

Statement	Yes	No
The number of DVDs sold declined by about 50% from 2008 through 2014.	<input type="radio"/>	<input type="radio"/>
The number of DVDs sold declined by the same amount each year from 2008 through 2014.	<input type="radio"/>	<input type="radio"/>
The slope of the line is the average yearly decline in DVDs sold from 2008 to 2014.	<input type="radio"/>	<input type="radio"/>

DVD SALES

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A line has been added to the graph to model these data points.

DVD Sales in the United Kingdom

Number of Years After 2008	Number of DVDs Sold (in Millions)
0	252.9
1	234.6
2	214.8
3	191.3
4	168.8
5	144.3
6	124.9

For the first item in this unit, students see a table containing three statements about DVD sales in the United Kingdom for the years 2008 through 2014, and they have to decide whether each statement is supported by the information shown in the graph. Note that the information from the introduction about how to interpret the data is repeated in the right panel, and that an additional paragraph was added to let students know about the line that was added to the graph. Also, the hover points are only active for the seven data points shown. That is, students cannot place the cursor anywhere along the line to get data, it is only available at the seven data points shown.

The first statement is supported by the data. Students can verify this statement in two ways: 1) compute the percent decline in DVD sales from 2008 to 2014 [i.e., $(252.9 - 124.9) \div 252.9$], an actual decline of 50.61%, which supports the statement of about a 50% decline in sales; or 2) compute the ratio of DVDs sold in 2014 to 2008 (i.e., $124.9 \div 252.9$) to see that the number of DVDs sold in 2014 is 49.39% of the number sold in 2008, which again is "about 50%" as specified in the statement.

The second statement is not supported by the data. Students need to look at the difference in the number of DVDs sold at each data point to see that that is not a true statement. However, four of the six differences show a very similar difference (between 18 and 19 million fewer DVDs sold), but between 2009 and 2010, the difference is 24.5 million, and between 2011 and 2012, the difference is 29.8 million fewer DVDs sold.

The third statement is supported by the data. Students can interpret that a linear model means a constant rate of change, which in this case because of the negative slope, implies that DVD sales are declining by the same average amount each year.

For this item, obtaining full credit is of moderate difficulty (Level 4) while it is relatively easy to obtain partial credit (Level 1a).

Unit Name – Item #	DVD Sales – CMA106Q01
Content Area	Uncertainty and Data
Process	Interpret/Evaluate
Context	Societal
Item Format	Complex Multiple Choice - Computer Scored
Answers	Full Credit (from top row to bottom row): Yes, No, Yes Partial Credit: any two selections correct
Estimated Difficulty	Level 4 (full credit) Level 1a (partial credit)

The DVD Sales Released Item #2

PISA 2022

DVD Sales
Question 2 / 3

Refer to "DVD Sales" on the right. Using the number keys, type your answer to the question.

The equation of the line is $d = 254 - 22n$, where d is the number of DVDs sold (in millions) and n is the number of years after 2008.

If this sales trend continues, in what year will the number of DVDs sold be less than 1 million according to the model?

Answer:

DVD SALES

The total number of DVDs sold each year in the United Kingdom from 2008 to 2014 is shown in the graph below.

The values on the horizontal axis represent the number of years **after** 2008. Place the cursor over the points in the graph to see the coordinates of that point. For example, the point (0, 252.9) indicates that there were 252.9 million DVDs sold in 2008. The point (1, 234.6) indicates that there were 234.6 million DVDs sold during the year 2009, etc.

A line has been added to the graph to model these data points.

DVD Sales in the United Kingdom

Number of Years After 2008	Number of DVDs Sold (in Millions)
0	252.9
1	234.6
2	216.3
3	198.0
4	179.7
5	161.4
6	143.1

For the second item in this unit, students are given the equation of the linear model and asked to use the model to estimate in what year the DVD sales first would fall below one million, a data point that is not shown on the graph. Note that students could again hover the cursor over the data points, but only at the seven data points shown. Using the given model, students can setup and solve an inequality such as $254 - 22n < 1$, which will lead to a solution of $n > 11.5$. However, since n represents years after 2008, students also have to convert this value into a year. In this scenario, 11 corresponds to the end of 2019, and 12 corresponds to the end of 2020, so the solution (11.5) from using the model implies that DVD sales would first drop below one million during the year 2020.

A full-credit response to this item is the year 2020. However, if a student responds with 11.5 (failing to correctly convert this number to 2020) or responds with 2019 (assuming they correctly calculated 11.5 but then incorrectly converted this to 2019 instead of 2020), then partial credit is given. Obtaining partial (Level 5) or full credit (Level 6) on this item is difficult.

Unit Name – Item #	DVD Sales – CMA106Q02
Content Area	Change and Relationships
Process	Formulate
Context	Societal
Item Format	Open Response - Computer Scored
Answers	Full Credit: 2020 Partial Credit: 2019 or 11.5
Estimated Difficulty	Level 6 (full credit) Level 5 (partial credit)

The DVD Sales Released Item #3

PISA 2022

DVD Sales
Question 3 / 3

Refer to "DVD Sales" on the right. Select from the drop-down menus to answer the question.

Since 1998, there have been several changes in the sales trends for the number of DVDs sold.

What are the sales trends and mathematical models that **best** fit these data for the year ranges 1998-2004 and 2005-2007?

Complete the table by selecting your answers from the drop-down menus. The last row has been completed for you as an example.

Years	Sales Trend	Mathematical Model
1998 – 2004	Select	Select
2005 – 2007	Select	Select
2008 – 2014	Decrease	Linear

DVD SALES

The total number of DVDs sold each year in the United Kingdom from 1998 to 2014 is shown in the graph. Place the cursor over the points in the graph to see the coordinates of that point.

DVD Sales in the United Kingdom

Number of DVDs Sold (in Millions)

Year

“Sales Trend” menu options: Increase or Decrease

“Mathematical Model” menu options: Linear or Non-linear

For the third item in this unit, students are shown a larger data set that shows DVD sales in the United Kingdom from 1998 through 2014. However, in this item, the independent variable for these data points is the actual year instead of the number of years after 2008. The data for the years 2008 through 2014 are the same data that are used in the first two items of this unit. Also, the 17 data points in this graph are hover points that display the year and the DVD sales for that year.

Students are given a table with the data set divided into smaller ranges of years, and they have to identify what type of sales trend (increase or decrease) and what type of mathematical model (linear or non-linear) best represent the data for the years specified. Since students had already been working with the data from 2008 through 2014 in the first two items of this unit, those answers were pre-populated in the table. Note that for the mathematical model, the goal was for students to be able to differentiate between data that can be reasonably-well modeled using a linear model and data that cannot be reasonably-well modeled using a linear model. Therefore, the general option of “non-linear” is used as opposed to specific types of non-linear models that would better fit the data for the years 1998 through 2004.

For this item it is moderately difficult (Level 3) to make all four correct selections. Partial credit was given for making both correct selections for any one range of years. Obtaining partial credit for this item was relatively easy (Level 1a). No credit is given if only one correct selection is made in each data range.

Unit Name – Item #	DVD Sales – CMA106Q03
Content Area	Change and Relationships
Process	Interpret/Evaluate
Context	Societal
Item Format	Complex Multiple Choice - Computer Scored
Answers	<p>Full Credit: 1998-2004: increase, non-linear 2005-2007: increase, linear</p> <p>Partial Credit: Correct selections only for 1998-2004 or Correct selections only for 2005-2007 or Any three selections correct</p>
Estimated Difficulty	Level 3 (full credit) Level 1a (partial credit)