

The   
**Nation's**  
Report Card

# Mathematics 2005

## Performance in Puerto Rico

**FOCUS ON THE CONTENT AREAS**

**National Assessment of Educational Progress**

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# What is The Nation's Report Card™?

The Nation's Report Card™ informs the public about the academic achievement of elementary and secondary students in the United States. Report cards communicate the findings of the National Assessment of Educational Progress (NAEP), a continuing and nationally representative measure of achievement in various subjects over time. The Nation's Report Card™ compares performance among states, urban districts, public and private schools, and student demographic groups.

For over three decades, NAEP assessments have been conducted periodically in reading, mathematics, science, writing, history, geography, and other subjects. By making objective information available on student performance at the national, state, and local levels, NAEP is an integral part of our nation's evaluation of the condition and progress of education. Only information related to academic achievement and relevant variables is collected. The privacy of

individual students is protected, and the identities of participating schools are not released.

NAEP is a congressionally mandated project of the National Center for Education Statistics within the Institute of Education Sciences of the U.S. Department of Education. The Commissioner of Education Statistics is responsible for carrying out the NAEP project. The National Assessment Governing Board oversees and sets policy for NAEP.

## Executive Summary

In 2005, public school students in Puerto Rico at grades 4 and 8 participated in a Spanish-language version of the National Assessment of Educational Progress in mathematics for the second time. Approximately 3,000 students in 100 public schools were assessed at each grade. Puerto Rico also participated in NAEP in 2003, but because changes were implemented for the 2005 administration, results for the two years cannot be compared.

Performance is summarized across the entire NAEP mathematics assessment and for five content areas on a 0–500 scale. Average scores for students in Puerto Rico at grades 4 and 8 for 2005 are presented in figures 1 and 2. Averages for public school students in the nation (excluding Puerto Rico) are shown for comparison. Because the scales were developed separately for each content area within each grade, direct comparisons should not be made between the average scores for the content areas. Question-level results presented in the report provide specific examples of student performance within each of the five areas. For each sample question, the percentage of correct responses is reported for Puerto Rico and the nation.

At grade 4, students in Puerto Rico scored lower, on average, than students in the nation overall and within each content area. There was no significant difference between the performance of male and female students in Puerto Rico overall. Female students in Puerto Rico scored higher than male students in geometry, but there was no difference between the performance of male and female students in the other content areas.

Students in Puerto Rico also scored lower at grade 8 than students in the nation overall and within each content area. There was no significant difference between the performance of male and female students in Puerto Rico overall. Female students in Puerto Rico scored higher than male students in the data analysis and probability content area, but there was no difference between the performance of male and female students in the other content areas.

Figure 1

Average fourth-grade NAEP mathematics scores in 2005, by content area

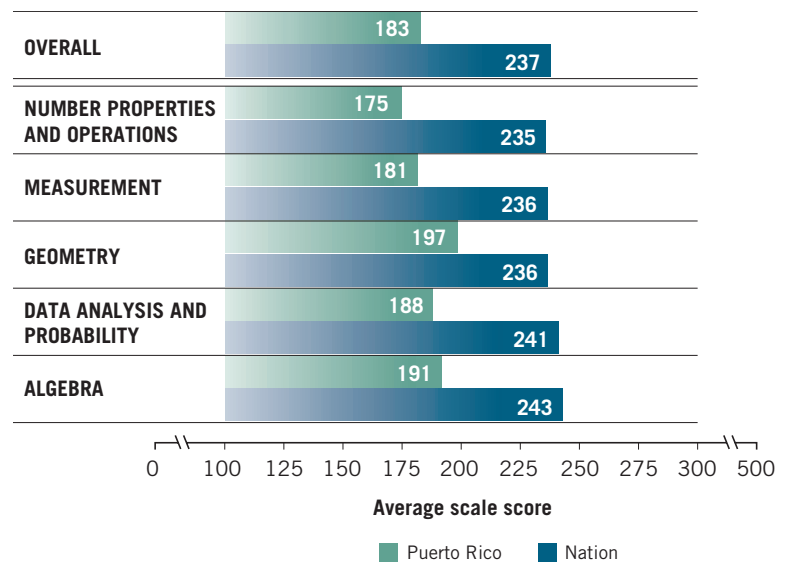
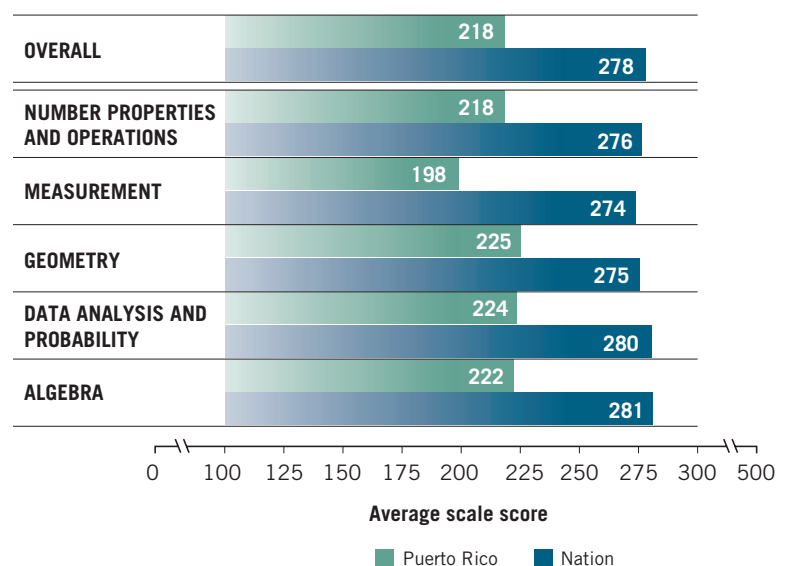


Figure 2

Average eighth-grade NAEP mathematics scores in 2005, by content area



SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

## The Mathematics Assessment

The content of all NAEP assessments is determined by subject-area frameworks that are developed by the National Assessment Governing Board in a comprehensive process involving a broad spectrum of interested parties, including teachers, curriculum specialists, subject-matter specialists, school administrators, parents, and members of the general public. Frameworks in NAEP are developed to survey student understanding over a broad range of content. The frameworks are available at <http://www.nagb.org/pubs/pubs.html/>.

The main content areas of the NAEP mathematics framework for grades 4 and 8 are

- number properties and operations,
- measurement,
- geometry,
- data analysis and probability, and
- algebra.

The NAEP mathematics framework specifies the percentage of questions to be assessed in each content area. The distribution of questions across content areas for grades 4 and 8 is shown in table 1.

Although some topics in these five content areas have changed across assessment years, the general focus of the mathematics assessments has remained consistent. In Puerto Rico, topics in the *Estándares de Excelencia Programa de Matemáticas* (Departamento de Educación 2000) are organized around the same five content areas as the *Mathematics Framework for the 2005 National Assessment of Educational Progress* (National Assessment Governing Board 2004).

The entire NAEP mathematics assessment consists of 10 sections of mathematics questions at each

grade. Each section includes 14 to 21 questions covering all five content areas. Because the assessment covers a breadth of content and includes more questions than any one student could reasonably answer, each student takes just a portion of the assessment, answering two sections of subject area questions. All of the data from the questions that students answer are combined to produce an average score for students in Puerto Rico.

Students are asked multiple-choice questions and constructed-response questions that require them to produce their own answers. Some questions at both grade levels incorporate the use of calculators, geometric shapes, rulers (at grade 4), or ruler/protractors (at grade 8). Geometric shapes are available for selected questions at both grade levels to help assess students' understanding of topics such as area and perimeter.

## Student Performance

The performance of public school students in Puerto Rico on a Spanish-language version of the NAEP mathematics assessment at grades 4 and 8 is documented in three reports. This report focuses on their performance in 2005 within each of the five mathematics content areas covered by the assessment. For comparison purposes, results are also shown for public school students in the nation (excluding Puerto Rico). A brief *Highlights* report presents average scores and achievement-level percentages for 2003 and 2005. A detailed *Technical Report* that describes the 2003 and 2005 assessments—including the translation of test questions into Spanish, the process of putting performance results onto the NAEP scale, and plans for future assessments in Puerto Rico—is forthcoming. All three reports may be accessed at [http://nationsreportcard.gov/puertorico\\_2005/](http://nationsreportcard.gov/puertorico_2005/).

Table 1

### Target percentage distribution of NAEP mathematics questions in 2005, by grade and content area

| CONTENT AREA                     | GRADE 4 | GRADE 8 |
|----------------------------------|---------|---------|
| Number properties and operations | 40      | 20      |
| Measurement                      | 20      | 15      |
| Geometry                         | 15      | 20      |
| Data analysis and probability    | 10      | 15      |
| Algebra                          | 15      | 30      |

SOURCE: U.S. Department of Education, National Assessment Governing Board, Mathematics Framework for the 2005 National Assessment of Educational Progress, 2004.

NAEP results are based on samples of student responses, and there is a margin of error associated with every result. Any differences that are mentioned in the text as “higher” or “lower” are statistically significant at the .05 level. Statistical significance is not marked in the figures and tables in the body of this report, but in all cases, the average scores and the percentages of correct or most complete responses for Puerto Rico were lower than those for the nation.

NAEP mathematics results are reported for grades 4 and 8 on a 0–500 scale. Scores for all students and for male and female students at each grade are presented in figures 3 and 4. At both grades, students in Puerto Rico scored lower, on average, than public school students in the nation overall.

Figure 3  
Average fourth-grade NAEP mathematics scores overall in 2005, by gender

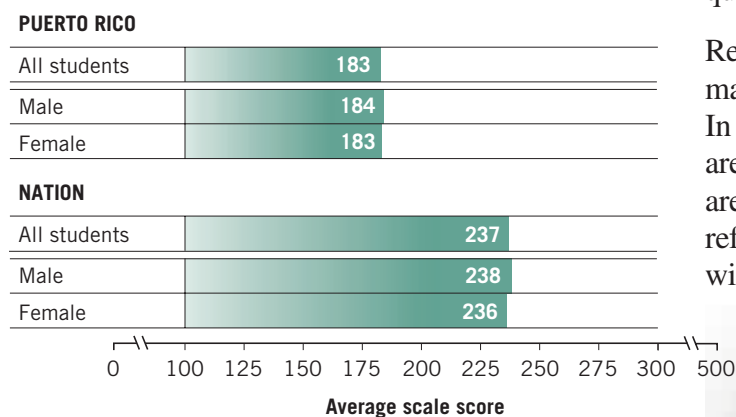
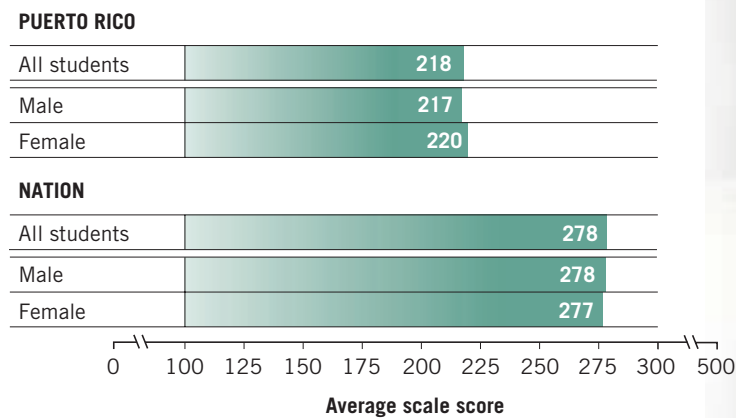


Figure 4  
Average eighth-grade NAEP mathematics scores overall in 2005, by gender

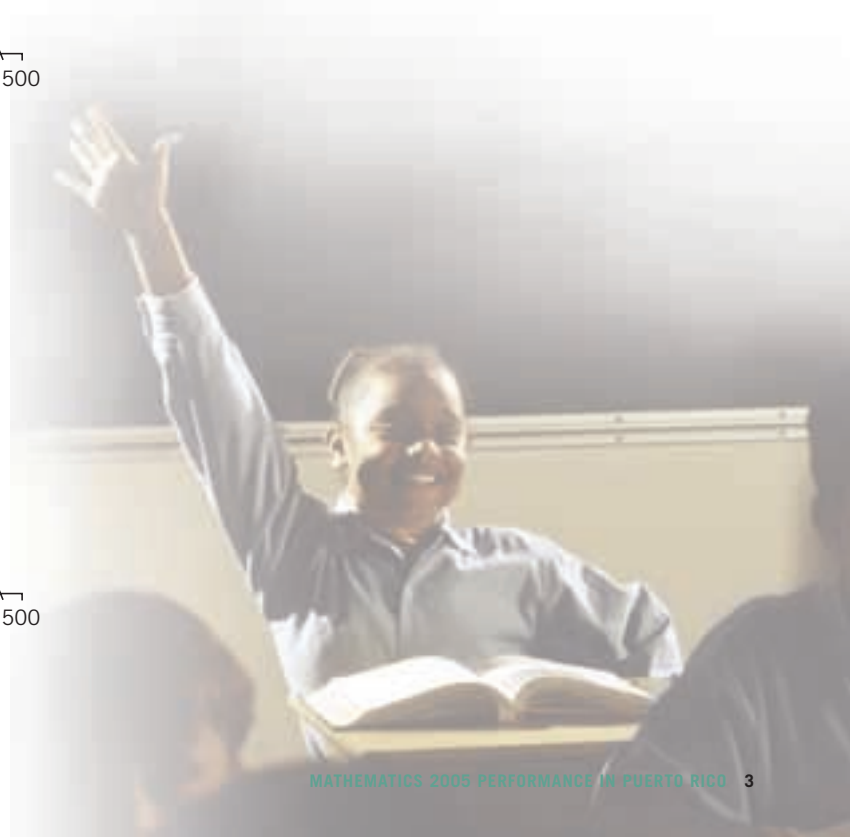


At grade 4, the scores for male and female students in Puerto Rico were not significantly different, while male students scored higher than female students in the nation. The same results for Puerto Rico and the nation were seen at grade 8.

The sections that follow provide more details about each of the five mathematics content areas. In addition to average scores, sample questions are shown as concrete examples of what students in Puerto Rico know and are able to do within selected subtopics. The percentages of students answering in each response category are presented in a table for each question. The row for the correct or most complete response is highlighted.

For a multiple-choice question, the response categories are the answer choices for the question. For a constructed-response question, the response categories are defined in the scoring guide for the question.

Results for selected questions from the 2005 NAEP mathematics assessment are listed in the appendix. In this appendix, average scores for the nation that are statistically different from those for Puerto Rico are marked with an asterisk. All of the questions referenced in this report are publicly available and will not be used in future assessments.



SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.



Photo by Norma Curet, Communications Office, Puerto Rico Department of Education

# Number Properties and Operations

Numbers are our main tools for describing the world quantitatively, so the ability to use number properties and operations is an important expectation of the 2005 NAEP mathematics framework. This content area focuses on students' understanding of ways to represent, calculate, and estimate numbers. In addition to basic computation skills, NAEP assesses students' ability to order and compare numbers and to solve problems in real-world settings using arithmetic operations.

At grade 4, students are expected to have a solid grasp of whole numbers and a beginning understanding of fractions. At grade 8, students are expected to be able to work with rational numbers (both fractions and decimals), ratios and proportional reasoning, scientific notation, and naturally occurring irrational numbers such as square roots and pi ( $\pi$ ).

**Subtopics in the number properties and operations content area are**

- Number sense
- Estimation
- Number operations
- Ratios and proportional reasoning
- Properties of number and operations

## Overall Results

At grade 4, the average score for students in Puerto Rico in the number properties and operations content area was 175. For Puerto Rico students at grade 8, the average scale score in this content area was 218. These scores were lower than the average scores for public school students in the nation.

In Puerto Rico, at both grades, the average score for male students was not significantly different from that of female students for the number properties and operations content area. For

students nationally at both grades, the average score of male students was higher than that of female students in this content area.

The next few pages contain sample questions from the number properties and operations content area in the 2005 NAEP mathematics assessment. These sample questions do not represent the entire range of content assessed in this content area and are for illustrative purposes only. For each sample question, the percentages of students answering in each response category are reported for both Puerto Rico and the nation. The row for correct or most complete response is highlighted.

Figure 5

**Average fourth-grade NAEP scores for number properties and operations in 2005, by gender**

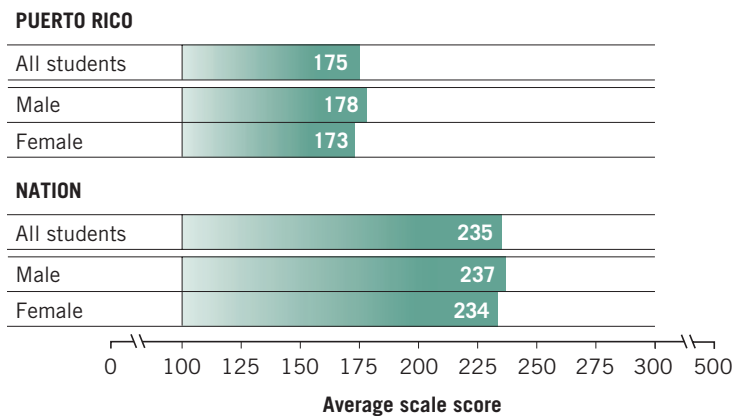
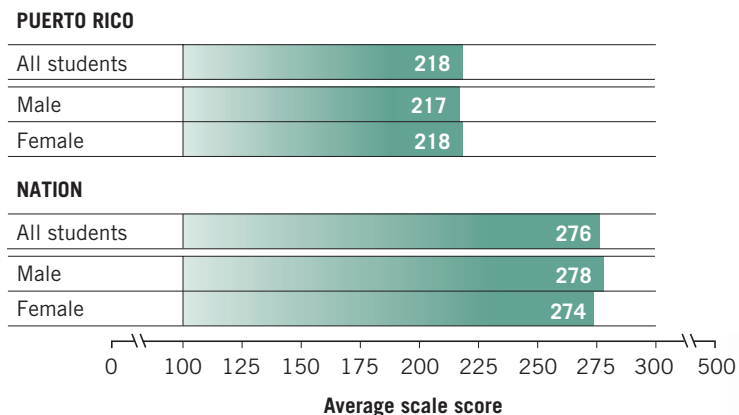
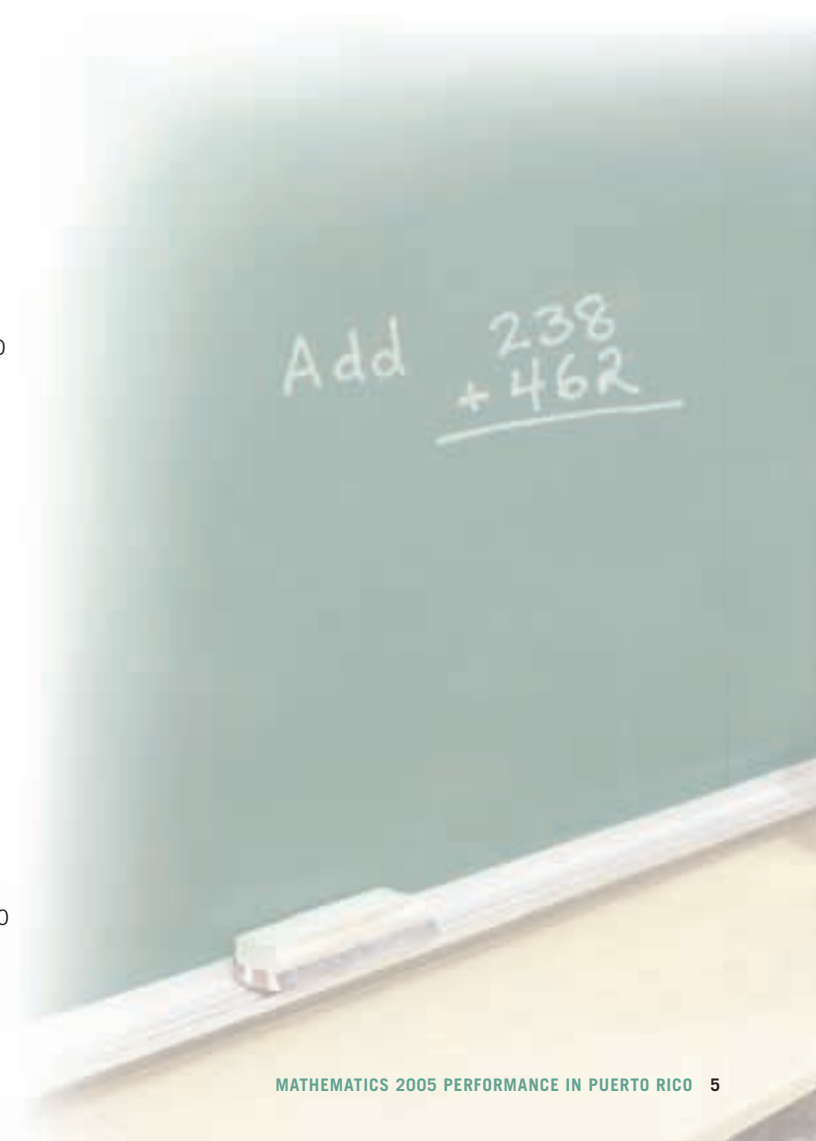


Figure 6

**Average eighth-grade NAEP scores for number properties and operations in 2005, by gender**



SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.



# Grade 4

Sample question 1 addresses the number sense subtopic, which includes questions about place value, ordering and comparing numbers, and using models and representations of numbers. Sample question 2 addresses the number operations subtopic, which includes questions about computation, the effects of operations on numbers, relationships between operations, and application problems involving numbers and operations.

## Sample Multiple-Choice Question

Sample question 1 asks the student to identify the standard representation of a number given in an expanded form. The framework objective measured in this question is “Compose or decompose whole quantities by place value.”

In Puerto Rico, 50 percent of the grade 4 students answered this question correctly. The incorrect answer choices for this question reflected place value errors in the hundreds, tens, or units place.

### Sample question 1

Which of these is equal to  $8,000 + 800 + 8$  ?

- (A) 8,088
- (B) 8,808
- (C) 8,880
- (D) 8,888

### Percentage of fourth-grade students in each response category

|                 | PUERTO RICO | NATION    |
|-----------------|-------------|-----------|
| Choice A        | 12          | 3         |
| <b>Choice B</b> | <b>50</b>   | <b>86</b> |
| Choice C        | 15          | 6         |
| Choice D        | 22          | 4         |
| Omit            | 1           | 1         |

NOTE: Detail may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.



## Sample Short Constructed-Response Question

Sample question 2 is an example of a computation question in a real-world setting. This is a multi-step problem that requires the student to find the number of tickets already sold (264) and then subtract this from the total number of tickets the club needs to sell (625). The framework objective measured in this question is “Solve application problems involving numbers and operations.”

This short constructed-response question was scored as either “Correct” or “Incorrect.” The correct response for this question is 361, and it was given by 6 percent of the grade 4 students in Puerto Rico.

A number of incorrect answers that were commonly given by students to this question were tracked during scoring. The correct and incorrect answers that were captured are described below:

- Correct** 361
- Incorrect #1** Incorrect answers other than those specified below
- Incorrect #2** 264, the total number of tickets already sold
- Incorrect #3** 889, the sum of the three numbers in the question
- Incorrect #4** 104 or 441 or 545, all of which result from subtracting two of the numbers in the question

### Percentage of fourth-grade students in each response category

|              | PUERTO RICO | NATION |
|--------------|-------------|--------|
| Correct      | 6           | 43     |
| Incorrect #1 | 65          | 38     |
| Incorrect #2 | 4           | 4      |
| Incorrect #3 | 4           | 3      |
| Incorrect #4 | 8           | 11     |
| Omit         | 12          | 2      |

NOTE: Detail may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

### Sample question 2

A club needs to sell 625 tickets. If it has already sold 184 tickets to adults and 80 tickets to children, how many more does it need to sell?

Answer: \_\_\_\_\_





# Grade 8

Sample question 3 addresses the ratios and proportional reasoning subtopic, which includes questions about ratios, proportions, and percents. Sample question 4 addresses the number sense subtopic, which includes questions about place value, ordering and comparing numbers, and translating between different representations of numbers. The emphasis of this subtopic is on rational numbers and includes scientific notation and absolute value.

## Sample Multiple-Choice Question

Sample question 3 asks students to solve a real-world problem using percent change. When determining a percent change, it is important to identify the “base” for computing the percent, which in this case is 90 employees. The framework objective measured in this question is “Solve problems involving percentages.”

In Puerto Rico, 11 percent of the grade 8 students answered this question correctly. Some misconceptions and errors represented by the incorrect answer choices to this question are given below:

- Finding 10 percent of 90 (choice A)
- Decreasing 90 by 10 percent (choice B)
- Increasing 90 by 1 percent (90.9) and rounding (choice C)
- Increasing 90 by 10 (choice E)

### Percentage of eighth-grade students in each response category

|                 | PUERTO RICO | NATION    |
|-----------------|-------------|-----------|
| Choice A        | 4           | 10        |
| Choice B        | 6           | 5         |
| Choice C        | 5           | 6         |
| <b>Choice D</b> | <b>11</b>   | <b>35</b> |
| Choice E        | 71          | 43        |
| Omit            | 3           | 1         |

NOTE: Detail may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

### Sample question 3

There were 90 employees in a company last year. This year the number of employees increased by 10 percent. How many employees are in the company this year?

- (A) 9
- (B) 81
- (C) 91
- 99
- (E) 100



## Sample Short Constructed-Response Question

Sample question 4 asks students to interpret a scale for rational numbers on a number line. The framework objective measured in this question is “Model or describe rational numbers or numerical relationships using number lines and diagrams.”

This short constructed-response question was scored as either “Correct” or “Incorrect.” The correct response for this question, 6.0, was given by 49 percent of the grade 8 students in Puerto Rico.

The specific incorrect responses that were tracked when this question was scored represent common student errors in interpreting the information displayed on the number line as described below:

- Correct** 6.0 or 6
- Incorrect #1** Incorrect answers other than those specified below
- Incorrect #2** 5.8 or 6.1, which is based on the misconception that each tick mark represents 0.1 of a unit either to the right of 5.6 or to the left of 6.2
- Incorrect #3** 5.10 (but not 5.1), which represents an understanding that each tick mark represents 0.2 of a unit, but does not correctly convert  $5.6 + 0.4$  to 6.0
- Incorrect #4** 58, 60, 61, or 62, which represent place value errors combined with a possible misinterpretation of the scale on the number line

### Percentage of eighth-grade students in each response category

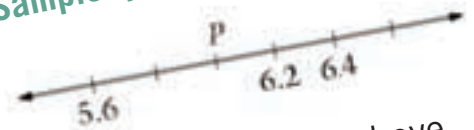
|              | PUERTO RICO | NATION |
|--------------|-------------|--------|
| Correct      | 49          | 88     |
| Incorrect #1 | 25          | 4      |
| Incorrect #2 | 12          | 4      |
| Incorrect #3 | 5           | 2      |
| Incorrect #4 | 1           | #      |
| Omit         | 8           | 1      |

# The estimate rounds to zero.

NOTE: Detail may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

### Sample question 4



On the number line above, what number would be located at point  $P$ ?

Answer: \_\_\_\_\_



Photo by Norma Curet, Communications Office, Puerto Rico Department of Education

# Measurement

**Measuring is the process by which numbers are assigned to describe the world quantitatively. The 2005 NAEP mathematics framework includes measurement attributes such as capacity, weight or mass, time, and temperature, as well as the geometric attributes of length, area, and volume.**

At grade 4, customary units such as inch, quart, pound, and hour, and common metric units such as centimeter, liter, and gram are emphasized, as well as the geometric attribute of length. At grade 8, the emphasis is on the use of square units for measuring area and surface area, cubic units for measuring volume, degrees for measuring angles, and rates. More emphasis is placed on area and angle measures than on linear measurements.

**Subtopics in the measurement content area are**

- **Measuring physical attributes**
- **Systems of measurement**

## Overall Results

At grade 4, the average score in the measurement content area for students in Puerto Rico was 181. At grade 8, the average score in measurement was 198 for students in Puerto Rico. These scores were lower, on average, than the scores for public school students in the nation.

While the national average score in the measurement content area was higher for male students than for female students at both grades, the average scores in measurement for male and female students in Puerto Rico were not significantly different.

The next few pages contain sample questions from the measurement content area in the 2005 NAEP mathematics assessment.

Figure 7  
Average fourth-grade NAEP scores for measurement in 2005, by gender

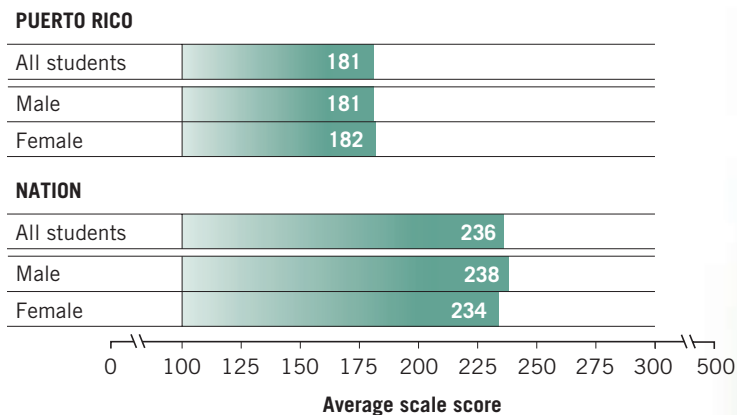
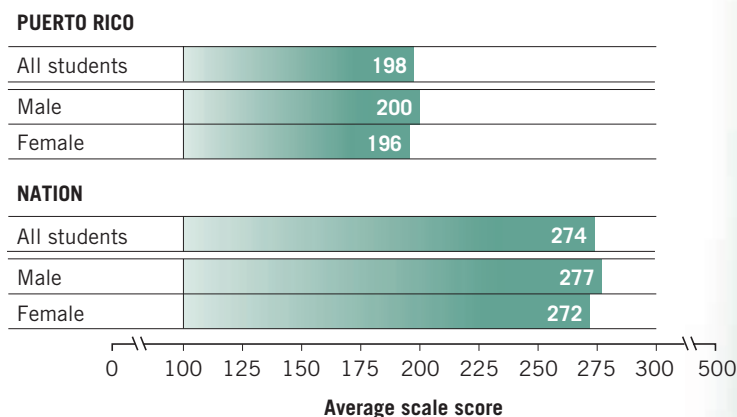


Figure 8  
Average eighth-grade NAEP scores for measurement in 2005, by gender



SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

The perimeter of a square is 36 inches. What is the length of one side of the square?

# Grade 4

Sample questions 5 and 6 address the measuring physical attributes subtopic. This subtopic includes questions about identifying attributes that can be measured; comparing objects or estimating the size of an object with respect to a given attribute such as length, time, or temperature; using appropriate measurement instruments; and solving problems involving the perimeter of plane figures or the area of squares and rectangles.

## Sample Multiple-Choice Question

Sample question 5 asks students to identify an attribute that can be measured with a meter stick. The framework objective measured in this question is “Identify the attribute that is appropriate to measure in a given situation.”

In Puerto Rico, 36 percent of the grade 4 students answered this question correctly.

The incorrect answer choices for this question are attributes of a swimming pool that could be measured using other tools (e.g., a thermometer or scale) or by counting.

### Sample question 5

Which of these could be measured using a meter stick?

- A The length of a swimming pool
- B The temperature of the water in a swimming pool
- C The weight of the water in a swimming pool
- D The number of people in a swimming pool

### Percentage of fourth-grade students in each response category

|          | PUERTO RICO | NATION |
|----------|-------------|--------|
| Choice A | 36          | 77     |
| Choice B | 19          | 11     |
| Choice C | 19          | 6      |
| Choice D | 25          | 5      |
| Omit     | 1           | 1      |

NOTE: Detail may not sum to totals because of rounding.

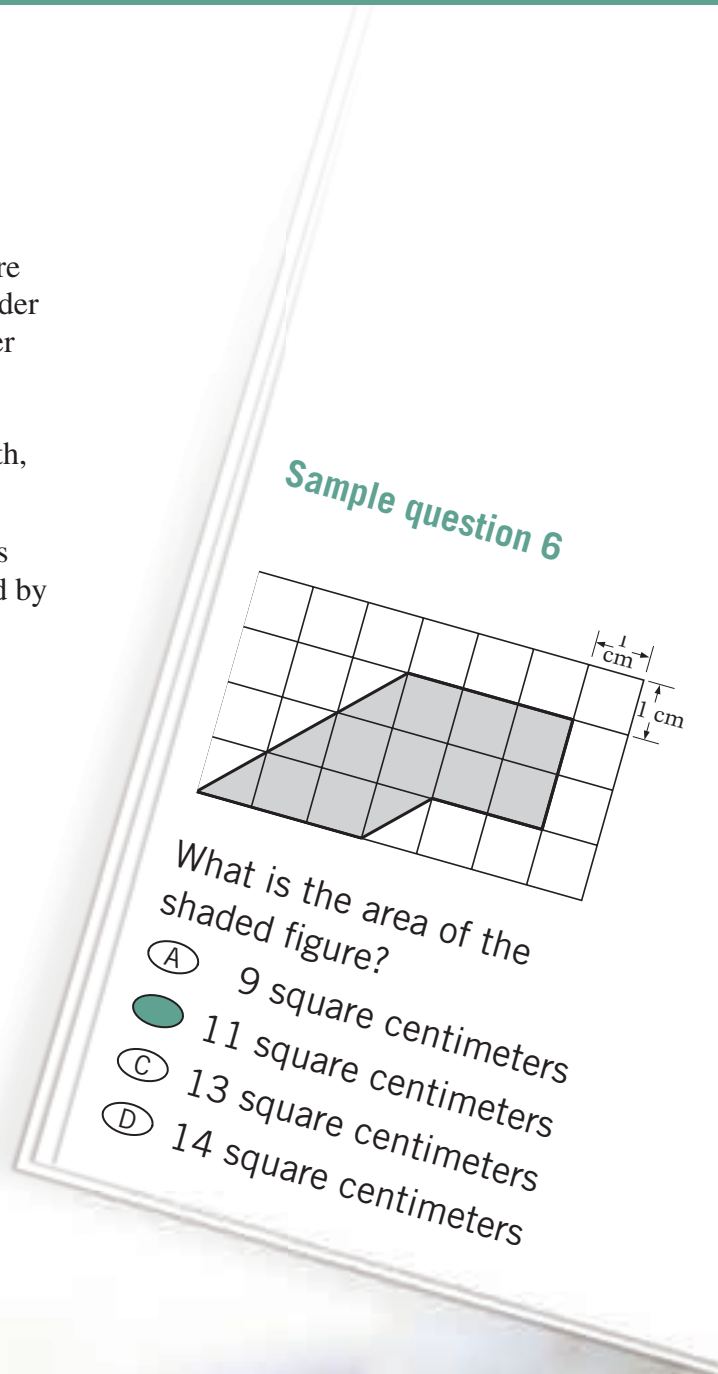
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

## Sample Multiple-Choice Question

Sample question 6 asks students to determine the area of a figure drawn on a grid. The correct solution requires students to consider both the number of squares that are fully shaded and the number of squares that are partially (one-half) shaded. The framework objective measured in this question is “Estimate the size of an object with respect to a given measurement attribute (e.g., length, perimeter, or area using a grid).”

In Puerto Rico, 17 percent of the grade 4 students answered this question correctly. Some misconceptions and errors represented by the incorrect answer choices in this question are given below:

- Counting only the squares that are fully shaded (choice A)
- Counting the total number of squares that are fully or partially shaded (choice C)
- Incorrectly counting the total number of squares that are fully or partially shaded (choice D)



### Percentage of fourth-grade students in each response category

|                 | PUERTO RICO | NATION    |
|-----------------|-------------|-----------|
| Choice A        | 25          | 20        |
| <b>Choice B</b> | <b>17</b>   | <b>47</b> |
| Choice C        | 46          | 25        |
| Choice D        | 9           | 7         |
| Omit            | 3           | 1         |

NOTE: Detail may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.



# Grade 8

Sample questions 7 and 8 address the measuring physical attributes subtopic. This subtopic includes questions about comparing objects or estimating the size of an object with respect to a measurement attribute such as length, angle, weight, or mass; using appropriate measurement instruments; solving problems involving the perimeter or area of plane figures; and solving problems involving the volume or surface area of solids.

## Sample Multiple-Choice Question

Sample question 7 asks students to find the length of a rectangular region given its perimeter and width. The framework objective measured in this question is “Solve mathematical or real-world problems involving perimeter or area of plane figures such as triangles, rectangles, circles, or composite figures.” In Puerto Rico, 21 percent of the grade 8 students answered this question correctly. Some misconceptions and errors represented by the incorrect answer choices in this question are given below:

- Dividing 390 by 75, which is the length of a rectangle that has an area of 390 square feet and a width of 75 feet (choice A)
- Dividing 390 by 4, which is the length of a side of a square that has a perimeter of 390 feet (choice B)
- Using a correct strategy with a subtraction error (choice D)
- Subtracting twice the width from the perimeter, which gives an answer that is twice the length of the rectangle (choice E)

### Percentage of eighth-grade students in each response category

|                 | PUERTO RICO | NATION    |
|-----------------|-------------|-----------|
| Choice A        | 9           | 19        |
| Choice B        | 23          | 11        |
| <b>Choice C</b> | <b>21</b>   | <b>39</b> |
| Choice D        | 18          | 9         |
| Choice E        | 27          | 19        |
| Omit            | 2           | 2         |

NOTE: Detail may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

### Sample question 7

A rectangular playground has a perimeter of 390 feet. The width of the playground is 75 feet. What is its length?

- (A) 5.2 feet
- (B) 97.5 feet
- (C) 120 feet
- (D) 130 feet
- (E) 240 feet



## Sample Short Constructed-Response Question

Sample question 8 is a short constructed-response question that asks students to draw a geometric figure meeting specified criteria. Students are provided with a ruler/protractor for this question. The framework objective measured in this question is “Select or use appropriate measurement instrument to determine or create a given length, area, volume, angle, weight, or mass.”

Student responses for this question were rated using the following three-level scoring guide:

**Correct** Response included a picture and frame drawn with the correct dimensions to a tolerance of  $\pm \frac{1}{8}$  inch on all measurements. (It was not necessary for the student to draw a picture of the sun and a flower, or anything else, in the inner rectangle in order to earn credit.)

**Partial** Response showed either a rectangular picture with correct dimensions and incorrect frame dimensions, or a rectangular picture with incorrect dimensions but one-inch frame drawn correctly.

**Incorrect** All incorrect responses.

In Puerto Rico, 16 percent of grade 8 student responses were rated “Correct.”

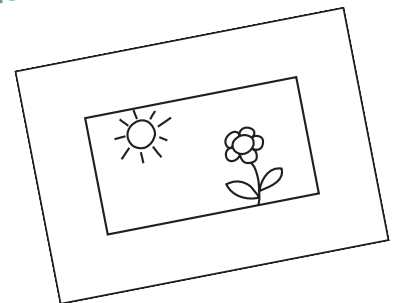
### Percentage of eighth-grade students in each response category

|           | PUERTO RICO | NATION |
|-----------|-------------|--------|
| Correct   | 16          | 43     |
| Partial   | 4           | 13     |
| Incorrect | 71          | 41     |
| Omit      | 8           | 2      |

NOTE: Detail may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

### Sample question 8



The figure above shows a picture and its frame.

In the space below, draw a rectangular picture 2 inches by 3 inches and draw a 1-inch-wide frame around it.



Photo by Norma Curet, Communications Office, Puerto Rico Department of Education

# Geometry

School geometry roughly mirrors the historical development of geometry, which began as a practical collection of rules for calculating lengths, areas, and volumes of common shapes. This expanded over time to include the study of the possible structures of space and ideas of symmetry and transformation.

Students at grade 4 are expected to be familiar with a library of simple figures and their attributes, both in a plane and in space. At grade 8, students are expected to be familiar with the properties of plane figures, especially parallel and perpendicular lines, angle relations in polygons, cross sections of solids, and the Pythagorean theorem.

Subtopics in the geometry content area are

- Dimension and shape
- Transformation of shapes and preservation of properties
- Relationships between geometric figures
- Position and direction
- Mathematical reasoning

## Overall Results

At grade 4, the average score in geometry for students in Puerto Rico was 197. At grade 8, the average score in geometry for Puerto Rico students was 225. These scores were lower, on average, than the scores for public school students in the nation.

In Puerto Rico at grade 4, the average score in geometry was higher for female students than for male students, while the results for students

in the nation showed no significant difference in the performance of male and female students in geometry. At grade 8, the average score in geometry for male students was not significantly different from that of female students in either Puerto Rico or the nation.

The next few pages contain sample questions from the geometry content area in the 2005 NAEP mathematics assessment.

Figure 9  
Average fourth-grade NAEP scores for geometry in 2005, by gender

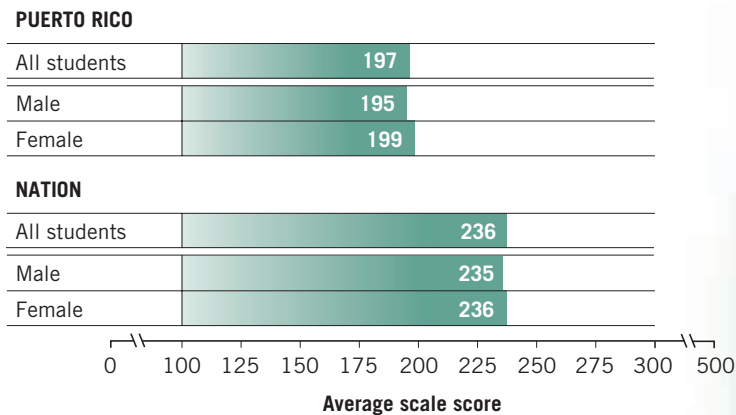
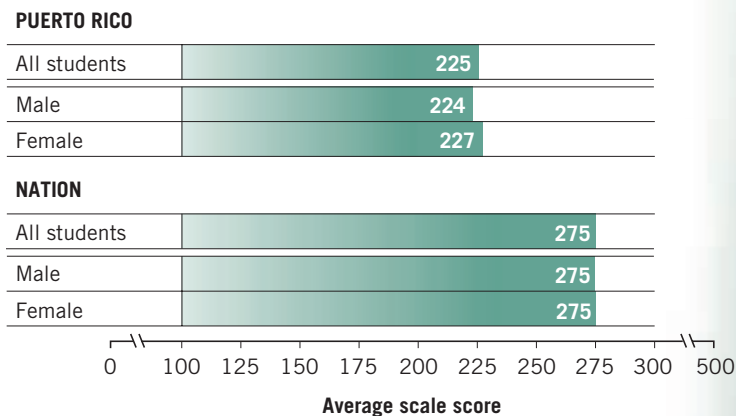
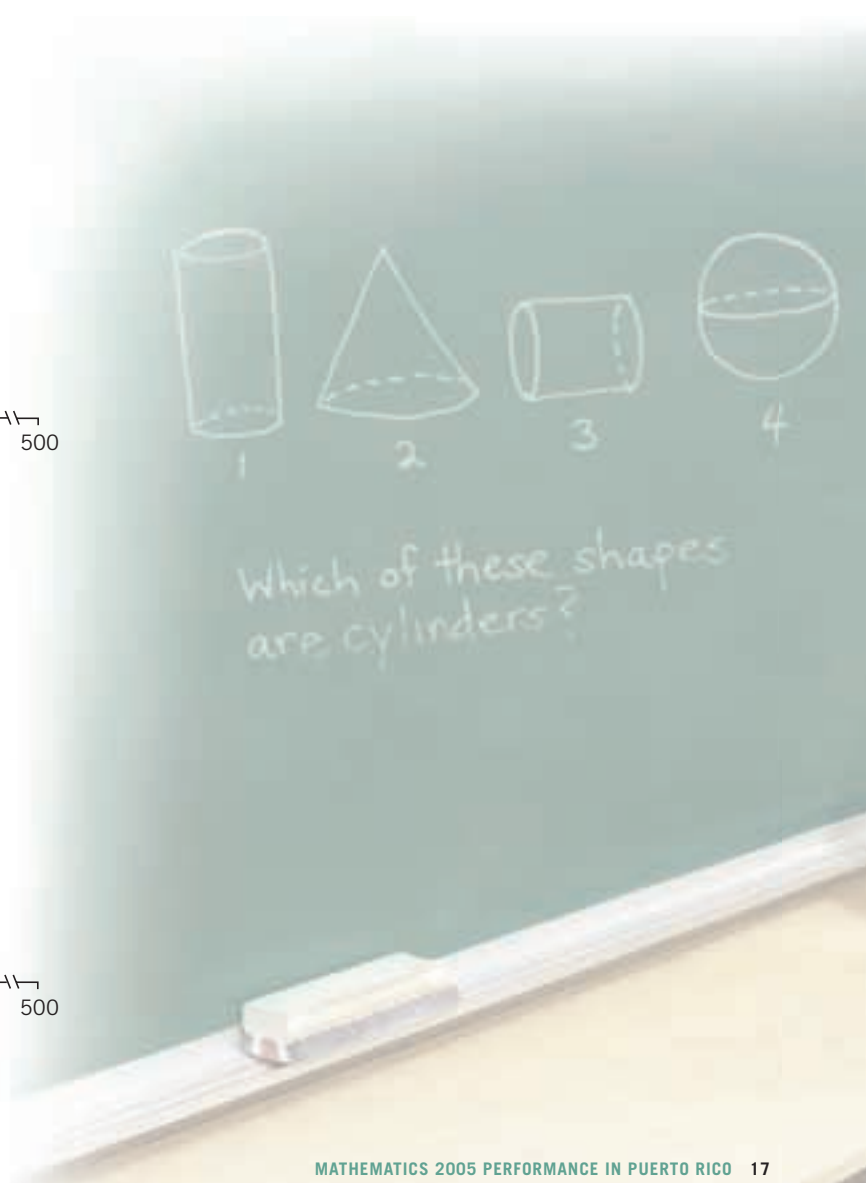


Figure 10  
Average eighth-grade NAEP scores for geometry in 2005, by gender



SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.



# Grade 4

Sample question 9 addresses the transformation of shapes and preservation of properties subtopic, which includes questions about symmetry, reflections, translations, and rotations, and how shapes change or remain the same under transformations. Sample question 10 addresses the relationships between geometric figures subtopic, which includes questions about patterns of geometric figures, properties of simple and compound figures, and two-dimensional faces of three-dimensional shapes.

## Sample Multiple-Choice Question

Sample question 9 involves visualizing the result of folding a two-dimensional figure into a three-dimensional form. The framework objective measured in this question is “Recognize which attributes (such as shape and area) change or don’t change when plane figures are cut up and rearranged.”

Sixteen percent of the grade 4 students in Puerto Rico answered this question correctly. The incorrect answer choices for this question represent misconceptions about the preservation of length and the relationship between the dimensions in the two figures. These incorrect choices reflect the following misconceptions and errors:

- Selecting other dimensions shown in one of the figures (choice A and choice D)
- Finding the difference in the lengths of the sides in the original rectangle (choice C)

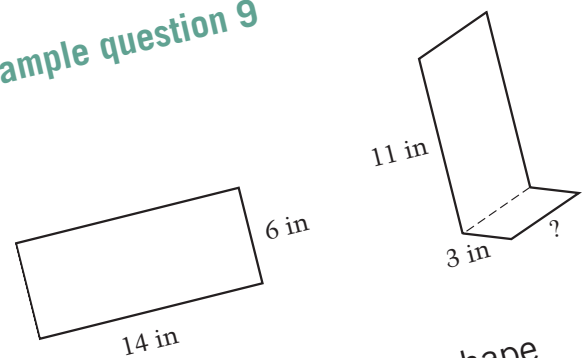
### Percentage of fourth-grade students in each response category

|                 | PUERTO RICO | NATION    |
|-----------------|-------------|-----------|
| Choice A        | 32          | 22        |
| <b>Choice B</b> | <b>16</b>   | <b>53</b> |
| Choice C        | 11          | 9         |
| Choice D        | 41          | 15        |
| Omit            | 1           | 1         |

NOTE: Detail may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

### Sample question 9



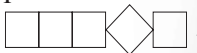
A piece of metal in the shape of a rectangle was folded as shown above. In the figure on the right, the “?” symbol represents what length?

- (A) 3 inches
- (B) 6 inches
- (C) 8 inches
- (D) 11 inches

## Sample Extended Constructed-Response Question

Sample question 10 is an extended constructed-response question that asks students to arrange five squares according to specified criteria. Students are provided with 10 paper squares to aid them in answering this question. The framework objective measured in this question is “Analyze or describe patterns of geometric figures by increasing number of sides, or changing size or orientation (e.g., polygons with more and more sides).”

Student responses for this question were rated using the following five-level scoring guide:

- Extended** Responses were those in which students drew three correct arrangements with lines shown separating the squares.
- Satisfactory** Responses were those in which students drew two correct arrangements with lines shown separating the squares.
- Partial** Responses were those in which students drew one correct arrangement with lines shown separating the squares or drew two of the figures given with a translation (must be translations or flips of two different figures).
- Minimal** Responses were those in which students made an attempt with five squares but showed no examples of a different way to arrange the squares (e.g., repeated one or more of the examples) or drew an arrangement with five squares that failed to meet criteria, such as .
- Incorrect** All incorrect responses.

In Puerto Rico, 2 percent of grade 4 student responses were rated “Extended.”

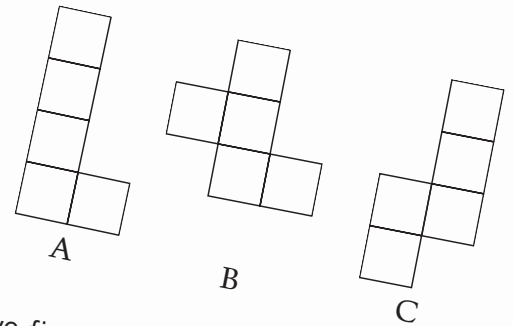
### Percentage of fourth-grade students in each response category

|              | PUERTO RICO | NATION |
|--------------|-------------|--------|
| Extended     | 2           | 11     |
| Satisfactory | 3           | 13     |
| Partial      | 13          | 20     |
| Minimal      | 23          | 19     |
| Incorrect    | 24          | 19     |
| Omit         | 36          | 17     |

NOTE: Detail may not sum to totals because of rounding.  
 SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

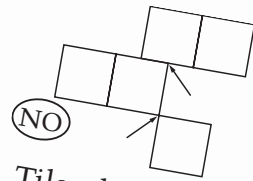
### Sample question 10

It is possible to arrange 5 tiles so that at least one side of each tile completely shares one side of another tile. Here are 3 different ways to do this.

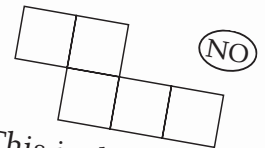


Two figures are not considered different if one figure can be turned or flipped to match the other.

The figures below are not examples of proper arrangements or new arrangements.



Tiles do not share whole sides.



This is the same as C turned.

Using 5 of your tiles, show 3 other different ways to arrange the tiles. Trace the tiles to show each figure. **Show the lines separating the individual squares.**



# Grade 8

Sample question 11 addresses the position and direction subtopic, which includes questions about relative positions of points and lines including midpoints, parallel and perpendicular lines, and points of intersection; cross sections of solids; and the representation of geometric figures in a rectangular coordinate plane. Sample question 12 addresses the dimension and shape subtopic, which includes questions about identifying, describing, and drawing two- and three-dimensional geometric objects.

## Sample Multiple-Choice Question

Sample question 11 asks students to find the midpoint of a line segment in the coordinate plane. The  $x$ -coordinate of the midpoint is the average of the  $x$ -coordinates of the endpoints, and the  $y$ -coordinate of the midpoint is the average of the  $y$ -coordinates of the endpoints. One way to find the average of two numbers is to find one-half of the sum of the two numbers. The framework objective measured in this question is “Describe relative positions of points and lines using the geometric ideas of midpoint, points on common line through a common point, parallelism, or perpendicularity.”

In Puerto Rico, 17 percent of the grade 8 students answered this question correctly. Some misconceptions represented by the incorrect answer choices in this question are given below:

- Finding one-half of the  $x$ -coordinate of the second point (4) and subtracting the  $x$ -coordinate of the first point (2), which is 2; similarly, finding one-half of the  $y$ -coordinate of the second point ( $4\frac{1}{2}$ ) and subtracting the  $x$ -coordinate of the first point (1), which is  $3\frac{1}{2}$  (choice A)
- Finding one-half of the positive difference of the  $x$ -coordinates and one-half of the positive difference of the  $y$ -coordinates (choice B)
- Finding the average of 1 and 8 for the  $x$ -coordinate of the midpoint, and finding the average of 2 and 9 for the  $y$ -coordinate of the midpoint (choice D)
- Finding the sum of the  $x$ -coordinates and the  $y$ -coordinates (choice E)

### Percentage of eighth-grade students in each response category

|                 | PUERTO RICO | NATION    |
|-----------------|-------------|-----------|
| Choice A        | 25          | 7         |
| Choice B        | 21          | 28        |
| <b>Choice C</b> | <b>17</b>   | <b>38</b> |
| Choice D        | 21          | 16        |
| Choice E        | 15          | 8         |
| Omit            | 2           | 3         |

NOTE: Detail may not sum to totals because of rounding.  
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

### Sample question 11

The endpoints of a line segment are the points with coordinates (2, 1) and (8, 9). What are the coordinates of the midpoint of this line segment?

- (A)  $(2, 3\frac{1}{2})$
- (B) (3, 4)
- (C) (5, 5)
- (D)  $(4\frac{1}{2}, 5\frac{1}{2})$
- (E) (10, 10)

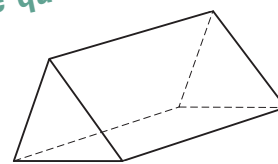
## Sample Multiple-Choice Question

Sample question 12 asks students to identify which of the given figures could be folded into a triangular prism. The framework objective measured in this question is “Demonstrate an understanding about the two- and three-dimensional shapes in our world through identifying, drawing, modeling, building, or taking apart.”

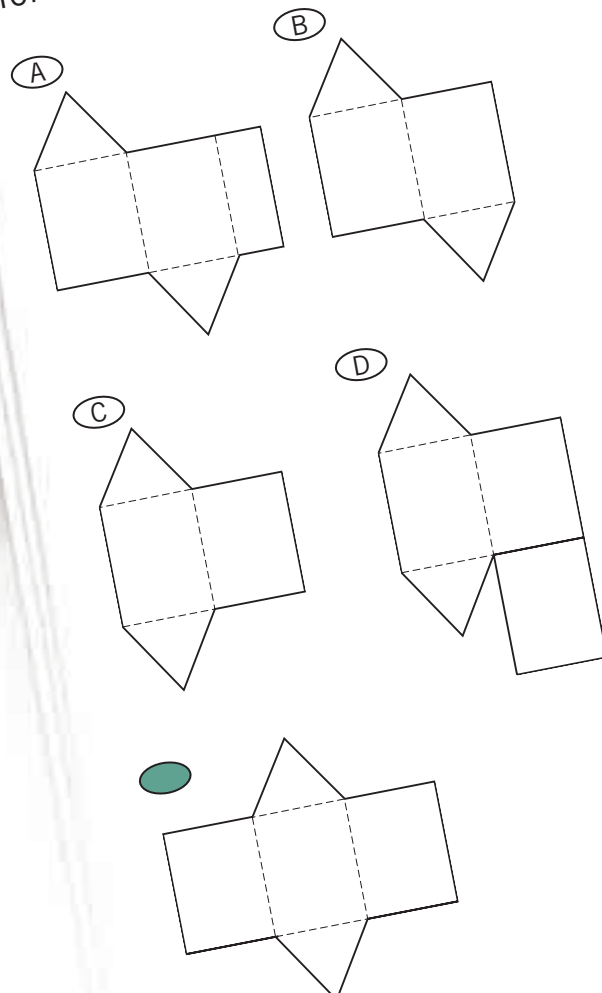
In Puerto Rico, 79 percent of the grade 8 students answered this question correctly. Some misconceptions represented by the incorrect answer choices in this question are given below:

- Not taking into account the scale of the figure when visualizing how it folds; one of the rectangular faces is not wide enough to completely meet all of its adjoining edges when folded (choice A)
- Not taking into account the total number of faces in the prism; in these instances, there are only four, instead of five, faces (choice B and choice C)
- Not taking into account the manner in which the faces need to be connected to fold up into the prism that is shown (choice D)

### Sample question 12



Which of the following can be folded to form the prism above?



### Percentage of eighth-grade students in each response category

|                 | PUERTO RICO | NATION    |
|-----------------|-------------|-----------|
| Choice A        | 5           | 4         |
| Choice B        | 5           | 4         |
| Choice C        | 6           | 2         |
| Choice D        | 3           | 2         |
| <b>Choice E</b> | <b>79</b>   | <b>87</b> |
| Omit            | 2           | #         |

# The estimate rounds to zero.

NOTE: Detail may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.



Photo by Norma Curet, Communications Office, Puerto Rico Department of Education

# Data Analysis and Probability

Data analysis is the process of collecting, organizing, summarizing, and interpreting data. This is the heart of the discipline called statistics. In the context of data analysis, probability can be thought of as the study of potential patterns in outcomes that have not yet been observed.

At grade 4, the data analysis and probability content area focuses on students' understanding of how data are collected and organized, how to read and interpret various representations of data, and basic concepts of probability. At grade 8, the emphasis is on students' ability to use a variety of techniques for organizing and summarizing data (including tables, charts, and graphs), to analyze statistical claims, and to use more formal terminology related to probability and data analysis.

Subtopics in the data analysis and probability content area are

- Data representation
- Characteristics of data sets
- Experiments and samples
- Probability



## Overall Results

At grade 4, the average score in data analysis and probability for students in Puerto Rico was 188. For Puerto Rico students at grade 8, the average score in data analysis and probability was 224. These scores were lower, on average, than the scores for public school students in the nation.

While the results for public school students in the nation at grade 4 showed a higher average score for male students than for female students in data analysis and probability, the results for Puerto Rico

students showed no significant difference between the average scores for male and female students in this content area. The pattern was different at grade 8, where in Puerto Rico, the average score in data analysis and probability was higher for female students than for male students, but there was no significant difference between the scores of male and female students in the nation.

The next few pages contain sample questions from the data analysis and probability content area in the 2005 NAEP mathematics assessment.

Figure 11  
Average fourth-grade NAEP scores for data analysis and probability in 2005, by gender

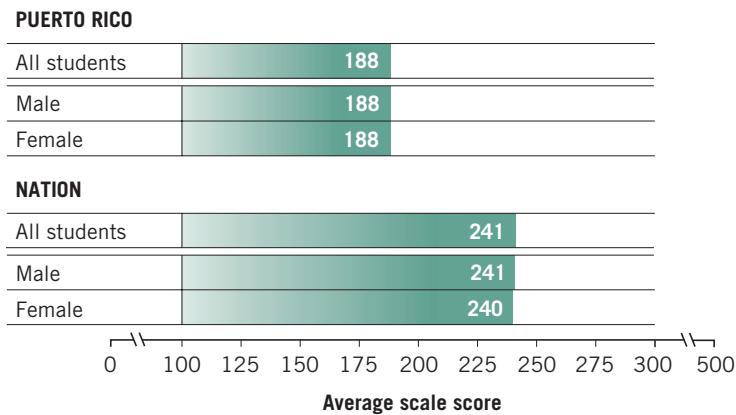
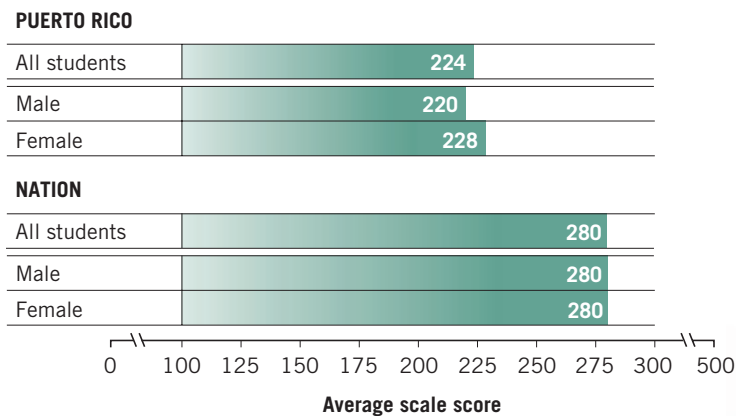
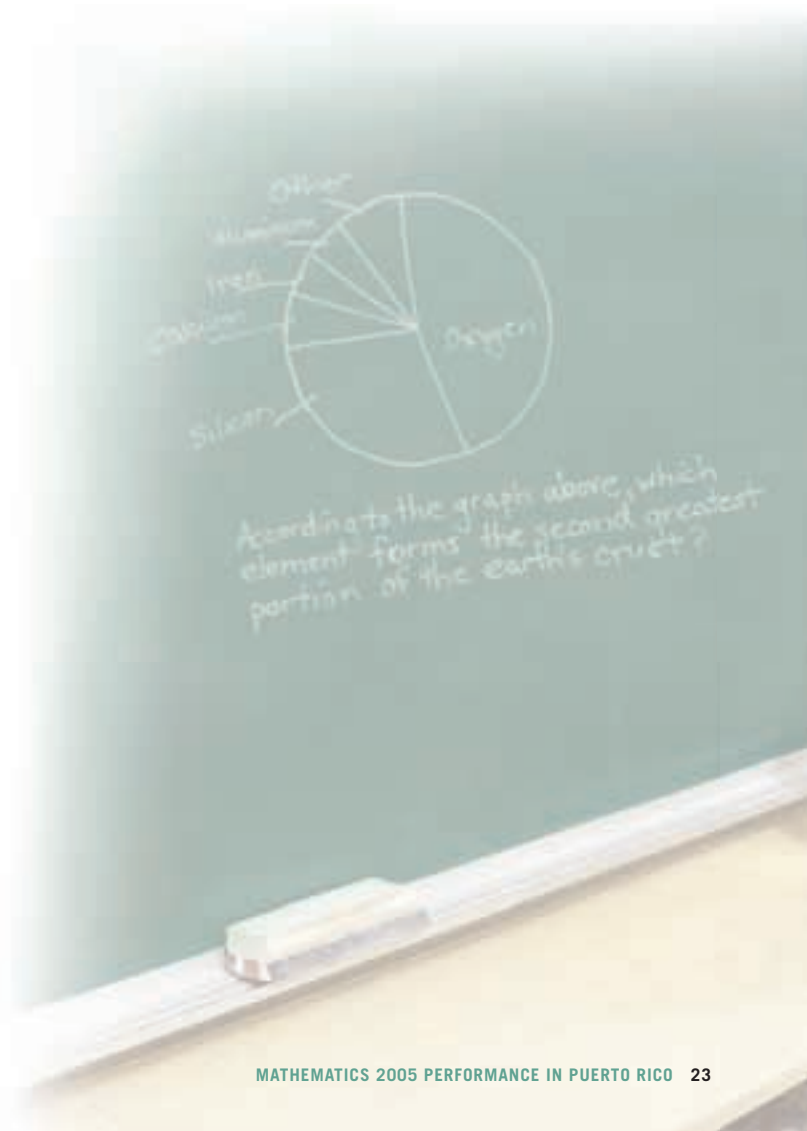


Figure 12  
Average eighth-grade NAEP scores for data analysis and probability in 2005, by gender



SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.



# Grade 4

Sample question 13 addresses the data representation subtopic, which includes questions about data presented in pictographs, bar graphs, circle graphs, line graphs, line plots, tables, and tallies. Sample question 14 addresses the probability subtopic, which includes questions about informal probabilistic thinking and counting or representing the outcomes of given events.

## Sample Multiple-Choice Question

Sample question 13 asks students to select an appropriate title for a bar graph. The framework objective measured in this question is “Read or interpret a single set of data.”

In Puerto Rico, 32 percent of the grade 4 students answered this question correctly. The following incorrect answer choices to this question are based on misinterpretations of what could be represented by each axis of the graph:

- The units on the scale are not appropriate (choice B).
- The number of categories does not match the number of bars on the graph (choice C and choice D).

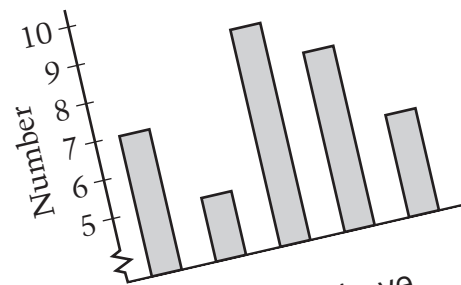
### Percentage of fourth-grade students in each response category

|          | PUERTO RICO | NATION |
|----------|-------------|--------|
| Choice A | 32          | 53     |
| Choice B | 17          | 20     |
| Choice C | 18          | 10     |
| Choice D | 32          | 16     |
| Omit     | 2           | 1      |

NOTE: Detail may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

### Sample question 13



Jin made the graph above. Which of these could be the title for the graph?

- Number of students who walked to school on Monday through Friday
- Number of dogs in five states
- Number of bottles collected by three students
- The number of students in ten clubs

## Sample Multiple-Choice Question

Sample question 14 asks students to identify the most likely outcome of a random event. The framework objective measured in this question is “Use informal probabilistic thinking to describe chance events (i.e., likely and unlikely, certain and impossible).”

In Puerto Rico, 57 percent of the grade 4 students answered this question correctly. The incorrect answer choices in this question are the other colors of the gumballs.

### Sample question 14

In a gumball machine there are 100 red, 75 blue, 50 green, and 125 yellow gumballs. These 350 gumballs are mixed up. Sam puts money in and one gumball comes out. Which color is most likely to come out?

- (A) Red
- (B) Blue
- (C) Green
- Yellow

### Percentage of fourth-grade students in each response category

|                 | PUERTO RICO | NATION    |
|-----------------|-------------|-----------|
| Choice A        | 17          | 4         |
| Choice B        | 11          | 2         |
| Choice C        | 15          | 3         |
| <b>Choice D</b> | <b>57</b>   | <b>91</b> |
| Omit            | 1           | #         |

# The estimate rounds to zero.

NOTE: Detail may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.





# Grade 8

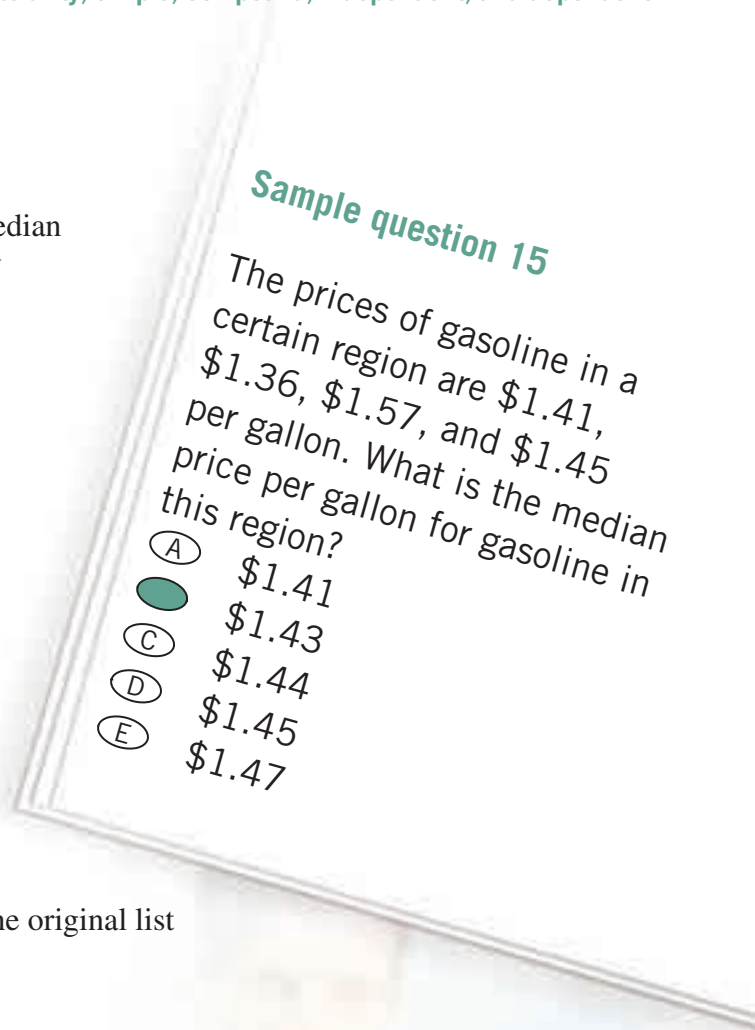
Sample question 15 addresses the characteristics of data sets subtopic, which includes questions about statistical measures that describe data sets, such as the mean, median, mode, range, interquartile range, and standard deviation; the effect of outliers; and scatterplots. Sample question 16 addresses the probability subtopic, which includes questions about theoretical and experimental probability; simple, compound, independent, and dependent events; and sample spaces.

## Sample Multiple-Choice Question

Sample question 15 asks students to determine the median of a data set. The median of four numbers is found by listing the numbers from least to greatest and then finding the average of the two middle numbers. The framework objective measured in this question is “Calculate, use, or interpret mean, median, mode, or range.”

In Puerto Rico, 15 percent of the grade 8 students answered this question correctly. The incorrect choices for this question reflect the following misconceptions and errors:

- Listing the numbers from least to greatest and then selecting one of the middle numbers (choice A and choice D)
- Using a correct strategy with an arithmetic error (choice C)
- Finding the average of the two middle numbers in the original list (choice E)



### Percentage of eighth-grade students in each response category

|                 | PUERTO RICO | NATION    |
|-----------------|-------------|-----------|
| Choice A        | 23          | 10        |
| <b>Choice B</b> | <b>15</b>   | <b>51</b> |
| Choice C        | 19          | 16        |
| Choice D        | 25          | 15        |
| Choice E        | 16          | 8         |
| Omit            | 2           | 1         |

NOTE: Detail may not sum to totals because of rounding.  
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

## Sample Short Constructed-Response Question

Sample question 16 is a short constructed-response question that requires students to reason about and explain the probability of a particular outcome for a problem in a real-world setting that involves dependent events. This question measures a student’s ability to analyze a situation that involves the probability of an event.

Student responses for this question were rated using the following three-level scoring guide:

**Correct** Responses presented a correct answer and explanation. For example:

Bill is incorrect because two of the candies that Bill ate were blue, which leaves only 8 blue candies. A total of 28 candies remain, which gives a probability of  $\frac{8}{28}$  (or  $\frac{4}{14}$  or  $\frac{2}{7}$ ) or 0.29.

**Partial** Responses showed some understanding of sampling without replacement (i.e., that the number of blue candies has decreased because he already removed two), but the explanation was not complete enough to earn full credit.

**Incorrect** All incorrect responses.

One percent of grade 8 student responses in Puerto Rico were rated as “correct.”

### Percentage of eighth-grade students in each response category

|           | PUERTO RICO | NATION |
|-----------|-------------|--------|
| Correct   | 1           | 17     |
| Partial   | 10          | 30     |
| Incorrect | 68          | 48     |
| Omit      | 21          | 4      |

NOTE: Detail may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

### Sample question 16

A package of candies contained only 10 red candies, 10 blue candies, and 10 green candies. Bill shook up the package, opened it, and started taking out one candy at a time and eating it. The first 2 candies he took out and ate were blue. Bill thinks the probability of getting a blue candy on his third try is  $\frac{10}{30}$  or  $\frac{1}{3}$ .

Is Bill correct or incorrect?

Explain your answer.



Photo by Norma Curet, Communications Office, Puerto Rico Department of Education

# Algebra

The concepts of algebra provide students with a powerful tool for understanding mathematics. A main focus in the study of algebra is representation, for example, using variables, functions, and coordinate geometry. Symbolic algebra easily and efficiently provides methods to represent problems and solve equations.

At grade 4, students are assessed in their understanding of algebraic representation, patterns, and rules; graphing points on a line or a grid; and the use of symbols to represent unknown quantities. Much of the emphasis of the algebra content area at grade 4 is on recognizing, describing, and extending patterns and rules. At grade 8, the emphasis is on students' understanding of patterns and functions; algebraic representations; algebraic expressions, equations, and inequalities; and linearity, including slope.

**Subtopics in the algebra content area are**

- Patterns, relations, and functions
- Algebraic representations
- Variables, expressions, and operations
- Equations and inequalities

## Overall Results

At grade 4, the average score in the algebra content area was 191 for students in Puerto Rico. At grade 8, the average score in algebra for Puerto Rico students was 222. These scores were lower, on average, than the scores for public school students in the nation.

The results by gender at grade 4 showed a higher average score in algebra for male students than for

female students in the nation. However, in Puerto Rico, there was no significant difference between the average scores in algebra for male and female students. At grade 8, there were no significant differences between male and female students' performance in algebra in Puerto Rico or in the nation.

The next few pages contain sample questions from the algebra content area in the 2005 NAEP mathematics assessment.

Figure 13  
Average fourth-grade NAEP scores for algebra in 2005, by gender

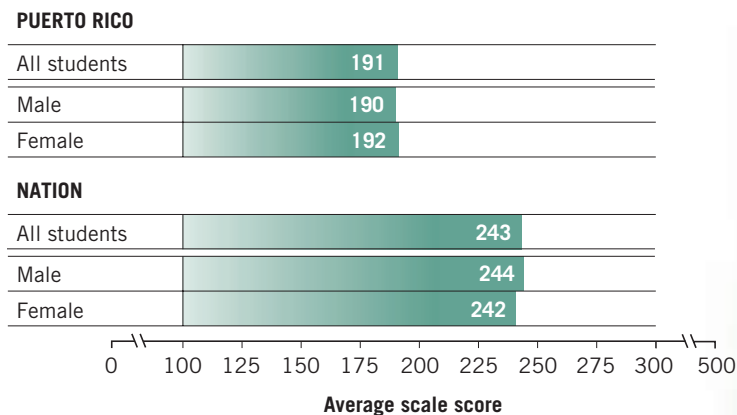
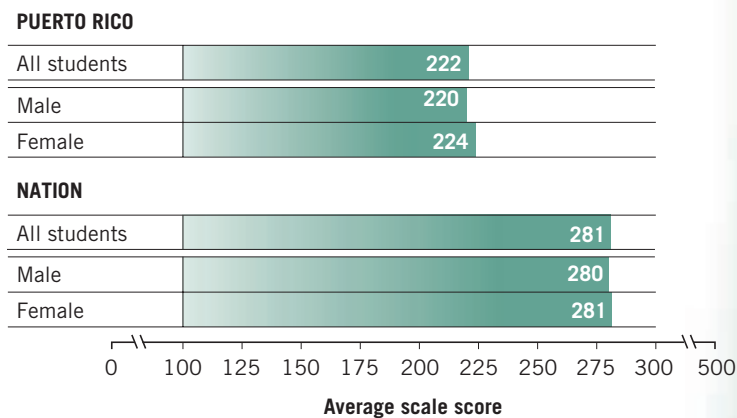
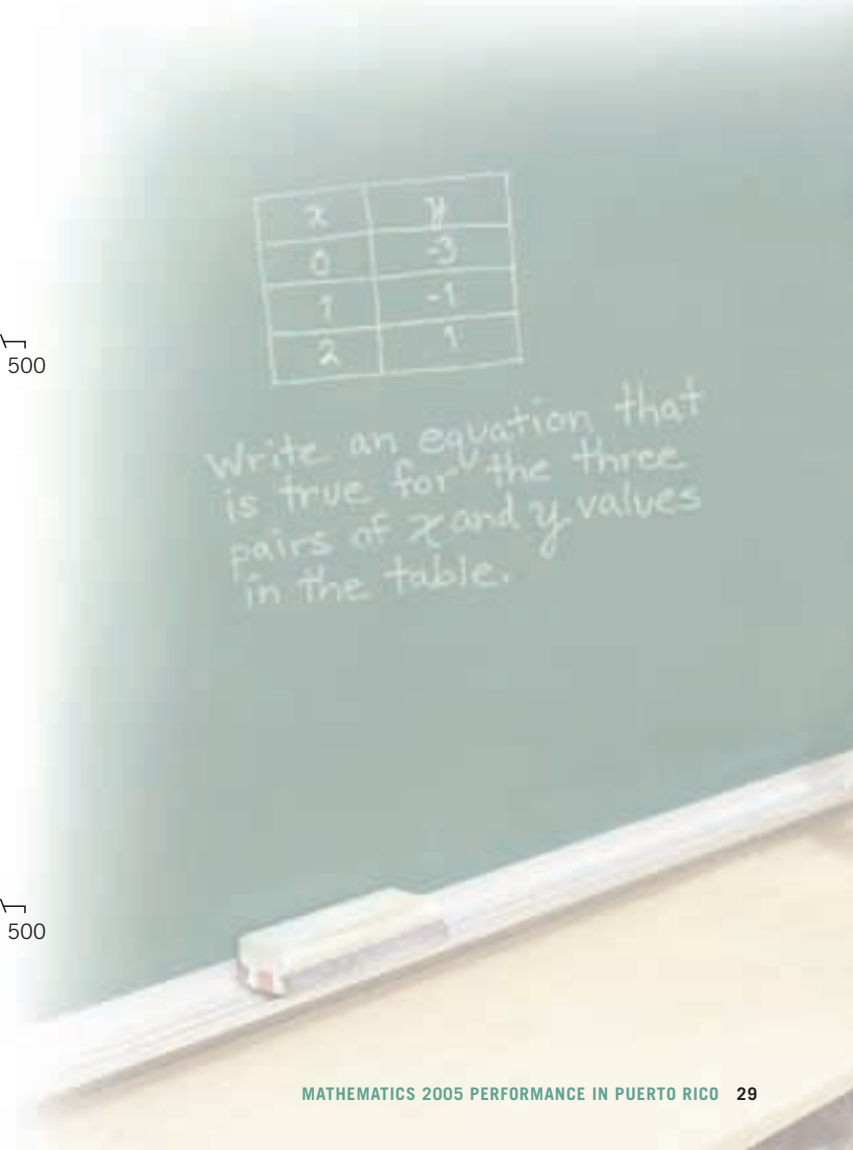


Figure 14  
Average eighth-grade NAEP scores for algebra in 2005, by gender



SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.



# Grade 4

Sample question 17 addresses the variables, expressions, and operations subtopic, which includes questions about representing unknown quantities with symbols and expressing simple mathematical relationships in number sentences. Sample question 18 addresses the patterns, relations, and functions subtopic, which includes questions about recognizing, describing, and extending patterns and rules.

## Sample Multiple-Choice Question

Sample question 17 asks students to identify the mathematical expression that represents a situation described in words. The framework objective measured in this question is “Use letters and symbols to represent an unknown quantity in a simple mathematical expression.”

In Puerto Rico, 30 percent of the grade 4 students answered this question correctly.

The incorrect answer choices for this question represent a misunderstanding of the mathematical relationship (multiplication) between the two quantities (hours per night and nights per week) given in the question.

### Percentage of fourth-grade students in each response category

|                 | PUERTO RICO | NATION    |
|-----------------|-------------|-----------|
| Choice A        | 39          | 26        |
| Choice B        | 12          | 4         |
| <b>Choice C</b> | <b>30</b>   | <b>61</b> |
| Choice D        | 17          | 8         |
| Omit            | 3           | 1         |

NOTE: Detail may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

### Sample question 17

$N$  stands for the number of hours of sleep Ken gets each night. Which of the following represents the number of hours of sleep Ken gets in 1 week?

- (A)  $N + 7$
- (B)  $N - 7$
- (C)  $N \times 7$
- (D)  $N \div 7$



## Sample Multiple-Choice Question

Sample question 18 requires students to identify a rule, given symbolically, that describes a pattern given in the table. The framework objective measured in this question is “Given a pattern or sequence, construct or explain a rule that can generate the terms of the pattern or sequence.”

In Puerto Rico, 15 percent of the grade 4 students answered this question correctly.

The incorrect answer choices represent the following misconceptions and errors:

- Finding a rule that describes only the relationship in the first row in the table (choice A)
- Recognizing that the number in the first column is doubled, but adding an incorrect constant value (choice B and choice D)

### Sample question 18

| $\square$ | $\triangle$ |
|-----------|-------------|
| 4         | 9           |
| 5         | 11          |
| 6         | 13          |
| 7         | 15          |

- Which rule describes the pattern shown in the table?
- (A)  $\square + 5 = \triangle$
- (B)  $\square + \square = \triangle$
- (C)  $\square + \square + 1 = \triangle$
- (D)  $\square + \square + 2 = \triangle$

### Percentage of fourth-grade students in each response category

|                 | PUERTO RICO | NATION    |
|-----------------|-------------|-----------|
| Choice A        | 53          | 43        |
| Choice B        | 15          | 20        |
| <b>Choice C</b> | <b>15</b>   | <b>24</b> |
| Choice D        | 12          | 11        |
| Omit            | 4           | 2         |

NOTE: Detail may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.



# Grade 8

Sample question 19 addresses the algebraic representations subtopic, which includes questions about analyzing, interpreting, and translating among different representations (symbolic, graphical, tabular, verbal, and pictorial) of a linear relationship; representing points in a rectangular coordinate system; and recognizing common nonlinear relationships in meaningful contexts. Sample question 20 addresses the equations and inequalities subtopic, which focuses on questions about linear equations and inequalities.

## Sample Multiple-Choice Question

Sample question 19 presents a graph in the  $xy$ -coordinate plane and requires students to estimate the  $x$ -intercept of the graph. The framework objective measured in this question is “Graph or interpret points that are represented by ordered pairs of numbers on a rectangular coordinate system.”

In Puerto Rico, 22 percent of the grade 8 students answered this question correctly.

Incorrect answer choices reflect the following misconceptions and errors:

- Finding the  $y$ -intercept (choice A)
- Misreading the scale or incorrectly estimating the  $x$ -coordinate (choice B, choice D, and choice E)

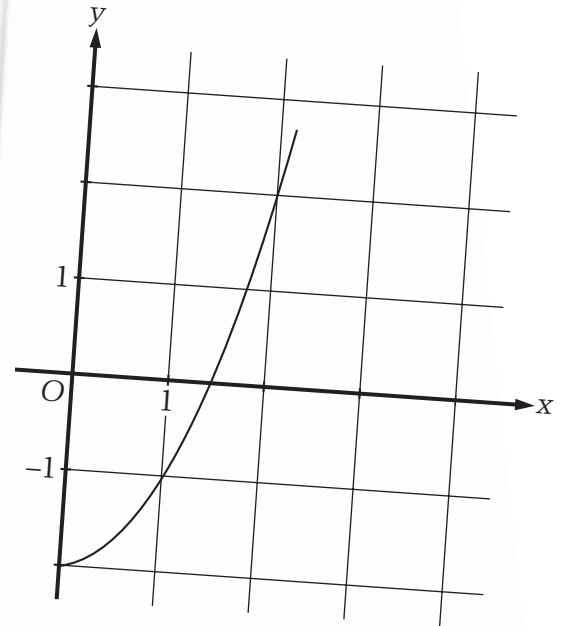
### Percentage of eighth-grade students in each response category

|                 | PUERTO RICO | NATION    |
|-----------------|-------------|-----------|
| Choice A        | 27          | 18        |
| Choice B        | 28          | 17        |
| <b>Choice C</b> | <b>22</b>   | <b>49</b> |
| Choice D        | 8           | 8         |
| Choice E        | 11          | 6         |
| Omit            | 3           | 1         |

NOTE: Detail may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

### Sample question 19



On the curve above, what is the best estimate of the value of  $x$  when  $y = 0$ ?

- (A) -2.0
- (B) 1.1
- (C) 1.4
- (D) 1.7
- (E) 1.9

## Sample Multiple-Choice Question

Sample question 20 asks students about the relationship between two variables when the value of one of the variables is changed. The framework objective measured in this question is “Interpret ‘=’ as an equivalence between two expressions and use this interpretation to solve problems.”

In Puerto Rico, 26 percent of the grade 8 students answered this question correctly. The incorrect answer choices represent misconceptions about the relationship between the expressions  $4x$  and  $4(x + 2)$ , which is equal to  $4x + 8$ .

### Sample question 20

In the equation  $y = 4x$ , if the value of  $x$  is increased by 2, what is the effect on the value of  $y$ ?

- It is 8 more than the original amount.
- (B) It is 6 more than the original amount.
- (C) It is 2 more than the original amount.
- (D) It is 16 times the original amount.
- (E) It is 8 times the original amount.

### Percentage of eighth-grade students in each response category

|          | PUERTO RICO | NATION |
|----------|-------------|--------|
| Choice A | 26          | 33     |
| Choice B | 23          | 9      |
| Choice C | 29          | 42     |
| Choice D | 9           | 4      |
| Choice E | 11          | 10     |
| Omit     | 2           | 2      |

NOTE: Detail may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

## Technical Notes and Data Appendix

Tables A-1 and A-2 list the NAEP 2005 mathematics questions for grades 4 and 8 that were released to the public after the assessment. They are organized by content area and increasing order of difficulty for students in Puerto Rico.

Student results are presented in terms of the average score for each question. An average score for a question is expressed as a fraction of the maximum possible score and ranges from 0.00 to 1.00.

The average score provides a way of comparing the difficulty of multiple-choice and constructed-response questions. For a multiple-choice question or a constructed-response question that is scored correct or incorrect, the average score is the percentage of correct responses expressed as a decimal. For a constructed-response question in which students could earn partial credit if they did not have a completely correct response, the average score is computed by adding the percent of students receiving full credit to a fraction of the percent of the students receiving partial credit.

An example of computing an average score for a constructed-response question can be provided for the grade 8 measurement question shown on page 15 that asks students to draw one rectangular region enclosed by another. Responses to this question were scored “Correct,” “Partial,” or “Incorrect.” For Puerto Rico, 16 percent of the students gave a fully correct answer, and an additional 4 percent of the students gave a partial answer. The average score for this question was computed as:  $16 + \frac{1}{2}(4) = 18$  or 0.18 when expressed as a decimal. The partial results were weighted by  $\frac{1}{2}$  because there were two levels of credit (correct and partial) for the question. Partial responses to a question with four levels of credit would receive weights of  $\frac{1}{4}$  (minimal),  $\frac{1}{2}$  (partial), and  $\frac{3}{4}$  (satisfactory). The fractions applied to partial responses are derived from the reciprocal of the number of credit levels for the question.

The average score for both multiple-choice and constructed-response questions takes into account those students who answered the question incorrectly, as well as those who reached the question but did not attempt to answer it. However, students who did not reach the question are not included in the calculation of an average score. (A student is considered as not reaching a question when neither that question nor any question following it in the section has been answered.)

For purposes of comparison, average scores for each question are also presented for public school students in the nation (excluding Puerto Rico). Significant differences between the average scores for a question for the nation and Puerto Rico are noted.

Table A-1. Average score on selected fourth-grade NAEP mathematics questions in 2005, by content area

|   | Puerto Rico |        |      | Puerto Rico |        |
|---|-------------|--------|------|-------------|--------|
|   | Rico        | Nation |      | Rico        | Nation |
| <b>Number properties and operations</b>                               |             |        |      |             |        |
| Fill in missing numbers on a number line                              | 0.51        | 0.77*  |      |             |        |
| <i>Identify a number given in expanded notation</i>                   | 0.50        | 0.86*  |      |             |        |
| <i>Solve a multi-step division problem</i>                            | 0.48        | 0.57*  |      |             |        |
| Given a solution, determine the numbers in the problem                | 0.41        | 0.68*  |      |             |        |
| <i>Subtract fractions with common denominators</i>                    | 0.36        | 0.53*  |      |             |        |
| Subtract two-digit number from three-digit number <sup>1</sup>        | 0.36        | 0.75*  |      |             |        |
| <i>Determine number of pieces from cutting wholes into fifths</i>     | 0.34        | 0.53*  |      |             |        |
| <i>Subtract two-digit numbers to solve a story problem</i>            | 0.22        | 0.79*  |      |             |        |
| <i>Identify number sentence matching a situation</i>                  | 0.22        | 0.33*  |      |             |        |
| Determine the value of a point on a number line <sup>1</sup>          | 0.17        | 0.55*  |      |             |        |
| <i>Identify numbers rounded to whole numbers<sup>1</sup></i>          | 0.14        | 0.45*  |      |             |        |
| Determine the greatest even number less than 20                       | 0.07        | 0.72*  |      |             |        |
| Solve a multi-step story problem <sup>1</sup>                         | 0.06        | 0.43*  |      |             |        |
| <b>Measurement</b>  |             |        |      |             |        |
| <i>Approximate fraction of an hour given minutes</i>                  | 0.37        | 0.49*  |      |             |        |
| <i>Determine which attribute could be measured with a meter stick</i> | 0.36        | 0.77*  |      |             |        |
| <i>Determine the area of shaded region on grid<sup>1</sup></i>        | 0.17        | 0.47*  |      |             |        |
| <b>Geometry</b>   |             |        |      |             |        |
| <i>Identify which shapes are cylinders<sup>1</sup></i>                | 0.69        | 0.87*  |      |             |        |
| <i>Determine how many angles are less than 90 degrees<sup>1</sup></i> | 0.18        | 0.44*  |      |             |        |
| Complete a set of written directions from a map                       | 0.17        | 0.67*  |      |             |        |
| Arrange tiles in different ways to satisfy given condition            | 0.16        | 0.36*  |      |             |        |
| <i>Determine the width of a rectangle after it is folded</i>          | 0.16        | 0.53*  |      |             |        |
| Mark a piece of paper to satisfy a given condition <sup>1</sup>       | 0.10        | 0.31*  |      |             |        |
| <b>Data analysis and probability</b>                                  |             |        |      |             |        |
| <i>Determine the most likely outcome in a story problem</i>           |             |        | 0.57 | 0.91*       |        |
| <i>Identify the appropriate title for a graph</i>                     |             |        | 0.32 | 0.53*       |        |
| Complete a bar graph from a description of data <sup>1</sup>          |             |        | 0.13 | 0.46*       |        |
| <b>Algebra</b>  |             |        |      |             |        |
| <i>Identify the missing figure in a pattern<sup>1</sup></i>           |             |        | 0.32 | 0.72*       |        |
| <i>Represent a situation with an algebraic expression</i>             |             |        | 0.30 | 0.61*       |        |
| Determine next number in given pattern                                |             |        | 0.23 | 0.68*       |        |
| <i>Identify equation to describe pattern given in table</i>           |             |        | 0.15 | 0.24*       |        |
| Determine missing numbers in number sentence                          |             |        | 0.10 | 0.44*       |        |
| Extend a pattern on a grid <sup>1</sup>                               |             |        | 0.08 | 0.38*       |        |

\* Significantly different ( $p < .05$ ) from Puerto Rico.

<sup>1</sup> Question was also administered to students at grade 8.

NOTE: Regular type denotes a constructed-response question. *Italic* type denotes a multiple-choice question. The average score for a question is expressed as a fraction of the maximum possible score and ranges from 0.00 to 1.00.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

Table A-2. Average score on selected eighth-grade NAEP mathematics questions in 2005, by content area

|   | Puerto Rico |       | Nation |  |
|---|-------------|-------|--------|--|
| <b>Number properties and operations</b>                                 |             |       |        |  |
| Subtract two-digit number from three-digit number <sup>1</sup>          | 0.73        | 0.88* |        |  |
| Determine the value of a point on a number line <sup>1</sup>            | 0.49        | 0.88* |        |  |
| Solve a multi-step story problem <sup>1</sup>                           | 0.43        | 0.76* |        |  |
| <i>Convert a written number to decimal form</i>                         | 0.38        | 0.67* |        |  |
| <i>Identify numbers rounded to whole numbers<sup>1</sup></i>            | 0.32        | 0.85* |        |  |
| <i>Identify piece of information not needed<sup>1</sup></i>             | 0.26        | 0.62* |        |  |
| Solve a story problem with multiple operations                          | 0.24        | 0.58* |        |  |
| <i>Recognize expanded form of number</i>                                | 0.24        | 0.40* |        |  |
| <i>Determine distance between points from a written description</i>     | 0.20        | 0.22  |        |  |
| Shade a fraction of a figure <sup>1</sup>                               | 0.16        | 0.72* |        |  |
| <i>Find the percent of a tip on a restaurant bill</i>                   | 0.15        | 0.29* |        |  |
| <i>Identify a point on a number line</i>                                | 0.13        | 0.41* |        |  |
| <i>Solve a story problem involving percent increase</i>                 | 0.11        | 0.35* |        |  |
| Write mathematical word problem given a scenario                        | 0.02        | 0.11* |        |  |
| <b>Measurement</b>  |             |       |        |  |
| <i>Determine the area of shaded region on grid<sup>1</sup></i>          | 0.34        | 0.77* |        |  |
| <i>List angle measures from smallest to largest</i>                     | 0.33        | 0.72* |        |  |
| Draw arrow to represent direction on a figure                           | 0.25        | 0.44* |        |  |
| <i>Find length of a rectangle given perimeter and width</i>             | 0.21        | 0.39* |        |  |
| Draw one rectangular region enclosed by another                         | 0.18        | 0.50* |        |  |
| <i>Calculate temperature increase on a thermometer</i>                  | 0.16        | 0.69* |        |  |
| Construct a figure on a grid  | 0.06        | 0.18* |        |  |
| Find area then construct another figure with same area                  | 0.02        | 0.26* |        |  |
| Determine how many boxes of tile are needed                             | 0.02        | 0.15* |        |  |
| <b>Geometry</b>   |             |       |        |  |
| <i>Identify which shapes are cylinders<sup>1</sup></i>                  | 0.90        | 0.93* |        |  |
| <i>Identify which figure can be folded to form a prism</i>              | 0.79        | 0.87* |        |  |
| Identify plane figure that results from opening a tube <sup>1</sup>     | 0.51        | 0.86* |        |  |
| Draw the reflection of a figure   | 0.50        | 0.77* |        |  |
| Shade a grid to form symmetric pattern                                  | 0.35        | 0.61* |        |  |
| <i>Find the measure of an acute angle</i>                               | 0.35        | 0.47* |        |  |
| <i>Determine how many angles are less than 90 degrees<sup>1</sup></i>   | 0.31        | 0.68* |        |  |
| <i>Determine how many colors are needed to paint a cube<sup>1</sup></i> | 0.28        | 0.79* |        |  |
| Mark a piece of paper to satisfy a given condition <sup>1</sup>         | 0.23        | 0.61* |        |  |
| <i>Determine which shape cannot be formed by 2 overlapping tiles</i>    | 0.21        | 0.53* |        |  |
| <i>Calculate midpoint of a line segment</i>                             | 0.17        | 0.38* |        |  |
| <i>Use properties of quadrilaterals to solve a problem</i>              | 0.14        | 0.24* |        |  |
| Construct a figure using tiles  | 0.08        | 0.34* |        |  |
| <b>Data analysis and probability</b>                                    |             |       |        |  |
| <i>Read and interpret a pie chart</i>                                   | 0.63        | 0.87* |        |  |
| Complete a graph given a set of data <sup>1</sup>                       | 0.59        | 0.90* |        |  |
| Complete a bar graph from a description of data <sup>1</sup>            | 0.59        | 0.79* |        |  |
| <i>Determine median price for a gallon of gasoline</i>                  | 0.15        | 0.51* |        |  |
| <i>Given probability, determine faces of a cube labeled R</i>           | 0.14        | 0.39* |        |  |
| Solve problem involving dependent events                                | 0.06        | 0.32* |        |  |
| <b>Algebra</b>  |             |       |        |  |
| <i>Determine the location of a city on a grid</i>                       | 0.66        | 0.85* |        |  |
| <i>Reason to order quantities<sup>1</sup></i>                           | 0.60        | 0.85* |        |  |
| <i>Identify the missing figure in a pattern<sup>1</sup></i>             | 0.56        | 0.89* |        |  |
| <i>Determine the 6th term in a pattern</i>                              | 0.34        | 0.59* |        |  |
| Extend a pattern on a grid <sup>1</sup>                                 | 0.28        | 0.63* |        |  |
| <i>Determine effect of increasing the value of one variable</i>         | 0.26        | 0.33* |        |  |
| Use graph of two linear equations to solve a problem                    | 0.24        | 0.44* |        |  |
| <i>Determine an equation given a table of x and y values</i>            | 0.23        | 0.53* |        |  |
| <i>Estimate the x-coordinate from the graph of a curve</i>              | 0.22        | 0.49* |        |  |
| <i>Determine coordinates to complete a rectangle</i>                    | 0.17        | 0.59* |        |  |
| <i>Identify the graph of whole numbers less than 5</i>                  | 0.17        | 0.36* |        |  |
| <i>Extend a pattern involving perfect squares</i>                       | 0.15        | 0.39* |        |  |
| <i>Identify an equivalent algebraic expression</i>                      | 0.15        | 0.42* |        |  |
| Reason about pattern on grid using concept of slope                     | 0.02        | 0.24* |        |  |

\* Significantly different ( $p < .05$ ) from Puerto Rico.

<sup>1</sup> Question was also administered to students at grade 4.

NOTE: Regular type denotes a constructed-response question. *Italic* type denotes a multiple-choice question. The average score for a question is expressed as a fraction of the maximum possible score and ranges from 0.00 to 1.00.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

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