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U. S. Department of Education
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Working Paper Series

Variance Estimation of Imputed Survey Data

Working Paper No. 98-14 October 1998

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1. Introduction

Missing data is a common problem in virtually all surveys. In cross-sectional surveys, missing data may mean no responses are obtained for a whole unit being surveyed (unit nonresponse), or that responses are obtained for some of the items for a unit but not for other items (item nonresponse). In panel or longitudinal surveys, the data may be missing in more complex patterns. For example, a unit may respond to one wave of a survey but not respond to other waves (wave nonresponse).

Unit and item nonresponse cause a variety of problems for survey analysts. Missing data can contribute to bias in the estimates and make the analyses harder to conduct and results harder to present. The most commonly used method for compensating for unit nonresponse in National Center for Education Statistics surveys is to adjust the weights of the respondents so that survey analysts can use the observed data to make inferences for the entire target population. The most frequently used method to compensate for item nonresponse in NCES surveys is imputation. Imputation consists of replacing the missing data item with a value that is either taken directly from a value reported by another respondent in the same survey or derived indirectly using a model that relates nonrespondents to respondents.

In practice, imputed values are often treated as if they were true values. This procedure is appropriate for developing estimates of totals, means, proportions, and most other estimates of first-order population quantities like quantiles, if the imputation does not cause serious systematic bias. However, to estimate the variance of these estimators when there is imputed data, it is no longer appropriate to use the standard formulae. As early as the 1950s, Hansen, Hurwitz, and Madow (1953) recognized that treating imputed values as observed values can lead to underestimating variances of these estimators if standard formulae are used. This underestimation may become more appreciable as the proportion of imputed items increases.

Analysts have developed a number of procedures to handle variance estimation of imputed survey data. In particular, Rubin (1987) proposed a multiple imputation procedure to estimate the variance due to imputation by replicating the process a number of times and estimating the between replicate variation. This multiple imputation procedure, however, may not lead to consistent variance estimators for stratified multistage surveys in the common situation of imputation cutting across sample clusters (Fay, 1991). Moreover, multiple imputation requires maintaining multiple complete data sets, which is operationally difficult, especially in large-scale surveys. More recently, Särndal (1992) outlined a number of model-assisted estimators of variance, while Rao and Shao (1992) proposed a technique that adjusts the imputed values to correct the usual or naive jackknife variance estimator for hot deck imputation. The Särndal and the Rao and Shao methods are appealing in that they yield unbiased variance estimators and only the imputed file (with the imputed fields flagged) is required for variance estimation. Kaufman (1996) proposed a variance estimation method similar to Särndal's method that can be used with a nearest neighbor imputation approach. Shao and Sitter (1996) proposed to perform an imputation procedure on each bootstrap sub-sample to

incorporate the imputation variability. This proposed bootstrap procedure is consistent irrespective of the sampling design, the imputation method, or the type of statistic used in inference. In fact, it is the only method that works without any restriction on the sampling design, the imputation method, or the type of statistic.

This research focuses on variance estimation and its consequences for analysts of NCES survey data. In section 2, Särndal's method, Kaufman's method, and Shao and Sitter's method are reviewed in more detail. In section 3, Shao and Sitter's bootstrap method is applied to the SASS 1993-94 Public School Teacher Survey component to assess the magnitude of imputation variance.

2. Literature Review

Three types of approaches to variance estimation in the presence of imputation are reviewed in this section: Särndal's model-assisted approach (Särndal, 1992). Kaufman's method that can be used when a nearest neighbor imputation approach is taken (Kaufman, 1996), and Shao and Sitter's bootstrap variance estimation method (Shao and Sitter, 1996). Shao and Sitter's method will also be applied to the SASS Public School Teacher component in the next section.

2.1 Särndal's Model-Assisted Method

Särndal (1992) proposed a model-assisted method. The model-assisted approach uses the fact that most imputation methods have an underlying model which justifies the imputed values. Let $U = \{1, \ldots, k, \ldots, N\}$ be the index set of a finite population. A probability sample s is selected from U with a given sampling design. Let r be the set of respondents, and nr the set of nonrespondents. The variable of interest is denoted by y. We are interested in the estimation of the population total $Y_U = \sum_U y_k$. The data after imputation consist of the values denoted $y_{\blacklozenge k}$, $k \in s$, such that

$$y_{\blacklozenge k} = \begin{cases} y_k, & if\, k \in r \\ y_{imp,k}, & if\, k \in nr \end{cases}$$

where y_k is an actually observed value, and $y_{imp,k}$ denotes the imputed value for the unit k. In the case of 100 percent response, $\hat{t} = \sum_{k \in s} w_k y_k$, where w_k is the weight given to the observation y_k. When the data contain imputations, the estimator of t is $\hat{t}_{\blacklozenge} = \sum_{k \in s} w_k y_{\blacklozenge k}$. The total error of $\hat{t}_{\blacklozenge}$ is decomposed as

$$\hat{t}_{\blacklozenge} - t = (\hat{t} - t) + (\hat{t}_{\blacklozenge} - \hat{t}) .$$

The mean squared error (MSE) of $\hat{t}_{\blacklozenge}$, denoted by V, is

$$V = E_p E_q (\hat{t}_{\blacklozenge} - t)^2 = V_{SAM} + V_{IMP} + 2V_{MIX} .$$

Here $p(\cdot)$ denotes the sampling design, that is, $p(s)$ is the known probability of realizing the sample s, $q(\cdot|s)$ the response mechanism, that is, $q(r|s)$ is the (unknown) conditional probability that the response set r is realized. It is assumed that $q(\cdot|s)$ is an unconfounded mechanism. That is, it may depend on the covariate values $\{x_k : k \in s\}$, but not on the

values $\{y_k : k \in s\}$ of the variable of interest. V_{SAM} is the sampling variance, V_{IMP} is the imputation variance, and V_{MIX} is the mixed term which measures the covariance between the sampling error and the imputation error. The components in the mean squared error (MSE) of $\hat{t}_{\blacklozenge}$ are hard to estimate unless a model for the relationship between x and y is brought in to assist the procedure. An example of such a model is:

$$y_k = \beta\ x_k + \varepsilon_k \qquad \text{for } k \in U.$$

The ε_k are random variables with $E_\xi(\varepsilon_k) = 0,\ \ E_\xi(\varepsilon_k^2) = \sigma^2 x_k$ and $E_\xi(\varepsilon_k \varepsilon_l) = 0$ for all $k \neq 1$ where E_ξ denotes expectation with respect to the model. The anticipated MSE (that is, the model expectation of the MSE) can be written as

$$E_\xi(V) = E_\xi(V_{SAM}) + E_p E_q\left[E_\xi\left\{(\hat{t}_{\blacklozenge} - \hat{t})^2 | s, r\right\}\right] + 2E_p E_q\left[E_\xi\left\{(\hat{t} - t)(\hat{t}_{\blacklozenge} - \hat{t}) | s, r\right\}\right].$$

The ξ-expectations appearing in the true variance components can be evaluated without difficulty, leading to expressions which depend on known x_k - values and on the unknown model parameters β and σ^2. The unconfoundedness of the nonresponse mechanism ensures that the order of expectation operators E_ξ and $E_p E_q$ can be reversed. Construct $\hat{V}_{SAM}$, $\hat{V}_{IMP}$, $\hat{V}_{MIX}$ such that

$$E_\xi\left\{E_p E_q(\hat{V}_{SAM}) - V_{SAM}\right\} = 0,$$

$$E_\xi\left\{E_p E_q(\hat{V}_{IMP}) - V_{IMP}\right\} = 0,$$

$$E_\xi\left\{E_p E_q(\hat{V}_{MIX}) - V_{MIX}\right\} = 0,$$

then

$$\hat{V} = \hat{V}_{SAM} + \hat{V}_{IMP} + 2\hat{V}_{MIX}$$

is anticipated to be unbiased for V. That is,

$$E_\xi\left\{E_p E_q(\hat{V}) - V\right\} = 0.$$

Särndal (1992), Lee, Rancourt, and Särndal (1995), and Rancourt, Särndal, and Lee (1994) applied this approach to four different imputation methods: respondent mean imputation, hot-deck imputation, ratio imputation, and nearest neighbor imputation.

With Särndal's method, the total variance can be estimated without multiple imputation but an explicit model for the relationship between auxiliary variable x and y is needed to assist the procedure. Therefore, if the imputation method is hard to model or if there is not enough evidence to assume a model for the relationship between y and x, this procedure is hard to implement. Also the unconfoundedness is satisfied often by assuming the response mechanism does not depend on the y-values, which is not always true.

2.2 Kaufman's Method

In practice, nearest neighbor imputation is often conducted in such a way that, within each imputation cell, sampling units are sorted so that two nearest neighbors can be identified for each missing case: one in ascending order and another in descending order, for example. Let *r* be the set of responding units and *nr* be the set of nonresponding units. Kaufman (1996) considered the following imputation set-up: for each $k \in nr$, one of the two nearest neighbors (donors) is randomly selected and assigned to the missing item. That is,

$$\tilde{y}_k = \begin{cases} y_k & \text{if } k \in r \\ y_{k1} I_k + y_{k2}(1 - I_k) & \text{if } k \in nr \end{cases}.$$

Here I_k is a random variable with $P(I_k = 1) = P(I_k = 0) = 0.5$, y_{k1} is the value of the first donor, y_{k2} is the value of the second donor, and y_k is the observed value for $k \in r$. Let $t_y = \sum_U y_k$ be the population total of variable *y*. If all sampled units are observed, an unbiased estimator of t_y is

$$\hat{y} = \sum_{k \in s} w_k y_k \ .$$

Here w_k is the design weight (inverse of inclusion probability). If the data are imputed and if we can assume the imputation does not cause very much systematic bias, then it is appropriate to use the customary estimator of t_y

$$\hat{y}_\bullet = \sum_{k \in s} w_k \tilde{y}_k \ .$$

In this section, we first derive the mean squared error of the underlying estimator $\hat{y}_\bullet$, denoted by $MSE(\hat{y}_\bullet)$, then we derive the variance of $\hat{y}_\bullet$, denoted by $V(\hat{y}_\bullet)$, both under Kaufman's imputation set-up. We also discuss Kaufman's approach of deriving $V(\hat{y}_\bullet)$. For the mean square error of $\hat{y}_\bullet$, notice

$$\begin{aligned} MSE(\hat{y}_\bullet) &= E(\hat{y}_\bullet - t_y)^2 \\ &= E(\hat{y}_\bullet - \hat{y} + \hat{y} - t_y)^2 \\ &= E(\hat{y}_\bullet - \hat{y})^2 + E(\hat{y} - t_y)^2 + 2E\left[(\hat{y}_\bullet - \hat{y})(\hat{y} - t_y)\right], \end{aligned}$$

and $E(\hat{y} - t_y)^2 = V(\hat{y})$, $COV\left[(\hat{y}_\bullet - \hat{y}), \hat{y}\right] = COV\left[(\hat{y}_\bullet - \hat{y}), (\hat{y} - t_y)\right] = E\left[(\hat{y}_\bullet - \hat{y})(\hat{y} - t_y)\right]$, hence

$$MSE(\hat{y}_\bullet) = E(\hat{y}_\bullet - \hat{y})^2 + V(\hat{y}) + 2COV\left[(\hat{y}_\bullet - \hat{y}), \hat{y}\right]. \qquad (1)$$

Here $E(\hat{y}_\bullet - \hat{y})^2$ is the imputation variance, $V(\hat{y})$ is the sampling variance, and $COV\left[(\hat{y}_\bullet - \hat{y}), \hat{y}\right]$ is the covariance between the sampling error and the imputation error. However, to estimate the components on the right-hand side of $MSE(\hat{y}_\bullet)$, we need an explicit model for the relationship between y and some auxiliary variables. Under Kaufman's imputation set-up, $COV\left[(\hat{y}_\bullet - \hat{y}), \hat{y}\right]$ can be written in a slightly different form. Notice

$$COV(\hat{y}_\bullet - \hat{y}, \hat{y}) = COV_1\left[E_I(\hat{y}_\bullet - \hat{y}), \hat{y}\right] + E_1\left[COV_I(\hat{y}_\bullet - \hat{y}, \hat{y})\right].$$

Here subscript 1 is with respect to sampling design and nonresponse mechanism, subscript *I* is with respect to donor selection. Also notice

$$\begin{aligned} E_I(\hat{y}_\bullet - \hat{y}) &= E_I(\hat{y}_\bullet) - \hat{y} \\ &= \sum_{k\in r} w_k y_k + 1/2 \sum_{k\in nr} w_k y_{k1} + 1/2 \sum_{k\in nr} w_k y_{k2} - \hat{y} \\ &= 1/2(\hat{y}_1 + \hat{y}_2) - \hat{y} \\ &= \bar{y}_\bullet - \hat{y}, \end{aligned}$$

here $\hat{y}_1 = \sum_{k\in r} w_k y_k + \sum_{k\in nr} w_k y_{k1}$, $\hat{y}_2 = \sum_{k\in r} w_k y_k + \sum_{k\in nr} w_k y_{k2}$, and

$$\begin{aligned} COV_I(\hat{y}_\bullet - \hat{y}, \hat{y}) &= E_I\left[(\hat{y}_\bullet - \hat{y})\hat{y}\right] - E_I(\hat{y}_\bullet - \hat{y})E_I(\hat{y}) \\ &= \left[E_I(\hat{y}_\bullet) - \hat{y}\right]\hat{y} - \left[E_I(\hat{y}_\bullet) - \hat{y}\right]\hat{y} \\ &= 0, \end{aligned}$$

so $COV(\hat{y}_\bullet - \hat{y}, \hat{y}) = COV_1\left[\bar{y}_\bullet - \hat{y}, \hat{y}\right]$. Therefore

$$MSE(\hat{y}_\bullet) = E(\hat{y}_\bullet - \hat{y})^2 + V(\hat{y}) + 2COV_1\left[(\bar{y}_\bullet - \hat{y}), \hat{y}\right]. \tag{2}$$

$MSE(\hat{y}_\bullet)$ can also be decomposed in the following way:

$$\begin{aligned} MSE(\hat{y}_\bullet) &= E(\hat{y}_\bullet - t_y)^2 \\ &= E(\hat{y}_\bullet - E(\hat{y}_\bullet) + E(\hat{y}_\bullet) - t_y)^2 \\ &= V(\hat{y}_\bullet) + \left[E(\hat{y}_\bullet) - t_y\right]^2. \end{aligned}$$

Here $V(\hat{y}_\bullet)$ is the variance of the underlying estimator $\hat{y}_\bullet$, and $E(\hat{y}_\bullet) - t_y$ is the bias of $\hat{y}_\bullet$. And $E(\hat{y}_\bullet) - t_y = E(\hat{y}_\bullet - \hat{y})$ can be estimated by an assisting model ξ in the following way. First evaluate the conditional expectation for given sample *s* and respondents *r*:

$$E_\xi(\hat{y}_\bullet - \hat{y} | s, r) = d_\xi.$$

Then for the given *s* and *r*, find a model unbiased estimator $\hat{d}_\xi$ for d_ξ. Here again we need a model and the assumption of unconfoundedness. The other component, $V(\hat{y}_\bullet)$, the variance of $\hat{y}_\bullet$, is obtained as following

$$V(\hat{y}_\bullet) = V(\hat{y}_\bullet - \hat{y} + \hat{y}) = V(\hat{y}_\bullet - \hat{y}) + V(\hat{y}) + 2COV\left[(\hat{y}_\bullet - \hat{y}), \hat{y}\right]. \tag{3}$$

Since $COV(\hat{y}_\bullet - \hat{y}, \hat{y}) = COV_1\left[\bar{y}_\bullet - \hat{y}, \hat{y}\right]$, $V(\hat{y}_\bullet)$ can be written in a slightly different form

$$V(\hat{y}_\bullet) = V(\hat{y}_\bullet - \hat{y}) + V(\hat{y}) + 2COV_1\left[(\bar{y}_\bullet - \hat{y}), \hat{y}\right]. \tag{4}$$

The right-hand side can be written in a form only with respect to the sampling design and response mechanism. Notice

$$V(\hat{y}) = V_1(\hat{y}),$$

$$V(\hat{y}_\bullet - \hat{y}) = V_1(\bar{y}_\bullet - \hat{y}) + E_1\left[1/4 \sum_{k\in nr} w_k^2 \left(y_{k1}^2 + y_{k2}^2\right)\right].$$

Therefore,

$$V(\hat{y}_\bullet) = V_1(\hat{y}) + V_1(\bar{y}_\bullet - \hat{y}) + E_1\left[1/4 \sum_{k\in nr} w_k^2 \left(y_{k1}^2 + y_{k2}^2\right)\right] + 2COV_1(\bar{y}_\bullet - \hat{y}, \hat{y}). \tag{5}$$

To estimate the variance components on the right side, however, we have to resort to an explicit model for the relationship between *y* and some auxiliary variables like Särndal's approach.

Another decomposition of $V(\hat{y}_\bullet)$ is simpler and probably more interesting. We decompose $V(\hat{y}_\bullet)$ into two parts: the sampling variance of the expected imputation value and the sampling expectation of the donor selection variance:

$$\begin{aligned} V(\hat{y}_\bullet) &= V_1 E_I(\hat{y}_\bullet) + E_1 V_I(\hat{y}_\bullet) \\ &= V_1\left[1/2(\hat{y}_1 + \hat{y}_2)\right] + E_1 V_I\left[\sum_{k\in nr} w_k\left(y_{k1} I_{k1} + y_{k2}(1 - I_k)\right)\right] \\ &= V_1(\bar{y}_\bullet) + E_1\left[1/4 \sum_{k\in nr} w_k^2\left(y_{k1}^2 + y_{k2}^2\right)\right]. \end{aligned} \tag{6}$$

Again, however, we need a model to estimate $V_1(\bar{y}_\bullet)$.

Kaufman (1996) took another approach to estimate $V(\hat{y}_\bullet)$. In Kaufman's method, a residual is defined for each $k \in s$:

$$\tilde{d}_k^R = \begin{cases} 0 & \text{if } k \in r \\ (2I_k - 1)\left(y_{j_k 2} - y_{j_k 1}\right) & \text{if } k \in nr \end{cases}$$

where j_k is a missing item within missing item *k*'s imputation cell and $\left(y_{j_k 2} - y_{j_k 1}\right)$ is the difference between the two nearest neighbors (donors) of j_k. Missing item j_k is selected independently from *k*'s imputation cell with known selection probability, for example, with selection probability proportional to design weights w_k. Then the residual is attached to $\tilde{y}_k$ to form another quantity $\hat{Y}$, which is used for the purpose of variance estimation:

$$\hat{Y} = \hat{y}_\bullet + \hat{d}^R = \sum_{k\in s} w_k\left(\tilde{y}_k + \tilde{d}_k^R\right).$$

The variance of $\hat{Y}$ contains variability from both $\hat{y}_\bullet$ and $\hat{d}^R$. It can be shown that (Theorem 1 of the appendix)

$$V\left(\hat{Y}\right) = V(\hat{y}) + V(\hat{y} - \hat{y}_\bullet) + 2COV_1(\bar{y}_\bullet - \hat{y}, \hat{y}) + E_1 V_2\left(\hat{d}^R\right).$$

Here $V_2\left(\hat{d}^R\right) = E_I V_R\left(\hat{d}^R\right) + V_I E_R\left(\hat{d}^R\right)$. According to formula (4) above or theorem 3 of the appendix, we have

$$V(\hat{y}_\bullet) = V\left(\hat{Y}\right) - E_1 V_2\left(\hat{d}^R\right). \tag{7}$$

Therefore, the variance of $\hat{y}_\bullet$ is the difference between $V\left(\hat{Y}\right)$ and $E_1 V_2\left(\hat{d}^R\right)$. Although the estimator of $E_1 V_2\left(\hat{d}^R\right)$ is often easy to find, the variance of $\hat{Y}$ is often hard to estimate, unless it can be shown that the same variance estimator for $\hat{y}$ can be used or an explicit model can help. Like Särndal's method, Kaufman's method does not require multiple imputation but the estimator for $V\left(\hat{Y}\right)$ may be hard to find and may need a model to assist the variance estimation. In addition, Kaufman's imputation method introduces a donor selection variance component into the total variance, which in turn inflates the total variance. Therefore, it is less efficient than Särndal's method. Nevertheless, this method leads to the same decomposition as formulae (4) and (3) (theorem 3 of the appendix).

2.3 Shao and Sitter's Method

Shao and Sitter (1996) proposed a bootstrap method for variance estimation of imputed data. Although they only proved that this method produces consistent bootstrap estimators for mean, ratio, or regression (deterministic or random) imputations under stratified multistage sampling, they believe that in fact the proposed bootstrap is the only method proposed thus far that works irrespective of the sampling design (single stage or multistage, simple random sampling or stratified sampling), the imputation method (random or nonrandom, proper or improper), or the type of estimator (smooth or nonsmooth). The method is paraphrased as following:

1) Draw a simple random sample $\left\{y_i^*: i = 1, \ldots, n\right\}$ with replacement from the original imputed data set $\mathbf{Y}_I = \left\{y_k : k \in A_R\,(respondents), \eta_k : k \in A_M\,(nonrespondents)\right\}$.
2) Let $\mathbf{Y}_R^* = \left\{y_i^* : i \in A_R^*\right\}$ and $\mathbf{Y}_M^* = \left\{y_i^* : i \in A_M^*\right\}$, where A_R^* and A_M^* denote the set of respondents and nonrespondents in the bootstrap sample. Apply the same imputation procedure used in constructing $\mathbf{Y}_I$ (using $\mathbf{Y}_R^*$ to impute $\mathbf{Y}_M^*$), and denote the bootstrap analog of $\mathbf{Y}_I$ by $\mathbf{Y}_I^*$.
3) Obtain the bootstrap analog $\hat{\theta}_I^* = \hat{\theta}\left(\mathbf{Y}_I^*\right)$ of $\hat{\theta}_I = \hat{\theta}\left(\mathbf{Y}_I\right)$, based on the imputed bootstrap data set $\mathbf{Y}_I^*$.
4) Repeat steps 1) - 3) *B* times. Apply Monte Carlo approximations to obtain bootstrap variance estimators for $\hat{\theta}_I$:

$$v\left(\hat{\theta}_I\right) \approx \frac{1}{B}\sum_{b=1}^{B}\left(\hat{\theta}^{*b} - \bar{\theta}^*\right)^2 ,$$

here $\bar{\theta}^* = B^{-1}\sum_{b=1}^{B}\hat{\theta}^{*b}$.

Shao and Sitter's method does not require any model or explicit variance formulae. Once the imputation procedure is programmed appropriately, Shao and Sitter's method is easy to implement. However, since *B* imputations should be performed for each item, extensive computation is required for large scale surveys. Maintaining the large amount of imputed data can be operationally difficult.

3. An Empirical Study

We chose Shao and Sitter's method to assess the magnitude of imputation variance in the SASS 1993-94 Public School Teacher Survey component based on the following considerations: 1) bootstrap method is used in SASS 1993-94 for variance estimation; 2) we do not have any reliable model on hand to allow us to perform Särndal's or Kaufman's method; 3) Kaufman's method nearest neighbor imputation has donor selection while the SASS 1993-94 imputation does not.

SASS 1993-94 Public School Teacher Survey data contains information on the 47,105 public school teachers who responded to the survey.

Four types of imputation methods are used in SASS 1993-94. They are (paraphrasing from Abramson et al., 1996, page 80):

(1) using data from other items of the same unit on the questionnaire;
(2) extracting data from a related component of SASS (for example, using data from a school record to impute missing values on the questionnaire for the LEA that operates the school);
(3) extracting data from the frame file (the information about the sample case from the sampling frame: the Private School Survey or the Common Core of Data);
(4) extracting data from the record for a sample case with similar characteristics ("hot deck ").

In this study, we investigated imputation method (4)—also called "stage 2 imputation." Methods (1), (2), and (3) are deductive imputation methods. In method (1), the imputed values are from other observed items of the same unit and in method (3) the imputed values are from the sampling frame file (PSS or CCD). For imputation method (2), the LEA's missing item is imputed through information from the sampled school which belongs to that LEA. According to Abramson et al. (1996), this type of imputation was performed only to the one-school LEAs. Therefore, the imputed values by methods (1), (2), or (3) are independent of the sample and the sample design. Assume the simplest response mechanism: respondents always respond and nonrespondents never respond. Then if the population is $\{y_1, y_2, \ldots, y_N\}$, the imputed values can be assumed to be $\{z_1, z_2, \ldots, z_N\}$. Here if y_k is actually observed, then $z_k = y_k$, otherwise z_k equals the value imputed by any method of (1), (2), or (3). Let $t_y = \sum_{k=1}^{N} y_k$ be the population total of *y*, $t_z = \sum_{k=1}^{N} z_k$ be the population total of *z*, and $\hat{t}_z = \sum_s z_k / \pi_k$ be the Horvitz-Thompson estimator of t_z (here π_k is the inclusion probability of unit k). We have the following decomposition

$$MSE\left(\hat{t}_z\right) = V\left(\hat{t}_z\right) + \left(t_z - t_y\right)^2 .$$

The first part, $V\left(\hat{t}_z\right)$, can be estimated by treating the imputed values as observed values while the second part is the bias of the imputation and assessing this bias is out of the scope of this study. If there is reason to believe the imputation bias is small, then treating the values imputed by any method of (1), (2), or (3) as observed values and using a standard variance estimation formula will not underestimate the variance. Or, if we can estimate the systematic bias caused by the imputation, the mean square error of the underlying estimator can then be estimated.

For method (4), however, the imputed data can not be treated as observed values. Actually every imputed value is a function of the sample, therefore the imputed values cannot be represented as a set of fixed values as $\{z_1, z_2, \ldots, z_N\}$.

SASS surveys are designed to produce reliable state estimates, and samples are selected systematically without replacement with large sampling rates within strata. To reflect the increase in precision due to large sampling rates, a without replacement bootstrap variance estimator procedure has been implemented for the 1993-94 SASS. Instead of drawing a simple random sample with replacement from the original sample, the bootstrap is done systematically without replacement with probability proportional to size as the original sampling was performed (Abramson et al., 1996).

In SASS 1993-94 components, 48 replicate weights were created to estimate variance using the bootstrap method. These replicate weights were subjected to various adjustments, including a sampling adjustment, a noninterview adjustment, and a ratio adjustment. In order to reflect these adjustments, these replicate weights should be used in the variance estimation. To this end, we used the Shao and Sitter's method in the following manner:

1) For each set of replicate weights $\{w_{ik}\}_{k=1,2,\ldots,n}$ (i = 1, 2, …, 48), cases with $w_{ik} = 0$ are dropped. Denote the remaining cases, which make up a bootstrap sub-sample, as $\mathbf{Y}_{Ii} = \{y_k : k \in A_{Ri}, \eta_k : k \in A_{Mi})\}$ (i = 1, 2, …, 48). This corresponds to Shao and Sitter's step 1.
2) Apply the same imputation method as was used to create the full sample imputation values and use $\{y_k : k \in A_{Ri}\}$ to impute $\{\eta_{ik}^* : k \in A_{Mi}\}$ (i = 1, 2, …, 48). This corresponds to Shao and Sitter's step 2. This re-imputed bootstrap sub-sample is denoted as s_i. That is
$$s_i = \{y_k : k \in A_{Ri}\} \cup \{\eta_{ik} : k \in A_{Mi}\}.$$
The missing values in the full sample are also imputed by using the nonmissing values in the full sample. This set of imputed values is denoted as
$$s_0 = \{y_k : k \in A_R\} \cup \{\eta_k^* : k \in A_M\}.$$
Thus, 48 sets of imputed bootstrap sub-samples and 1 set of imputed full sample are obtained.
3) Calculate the $\hat{\theta}_i$ of interest from s_i, weighted by replicate weights $\{w_{ik}\}$ $(i = 1, \ldots 48)$, and the $\hat{\theta}$ from full sample s_0, weighted by the full sample weight $\{w_k\}$. The variance of $\hat{\theta}$ is estimated by
$$v(\hat{\theta}) = \frac{1}{48}\sum_{i=1}^{48}\left(\hat{\theta}_i - \hat{\theta}\right)^2 .$$

Another difference between the variance estimator we used above and Shao-Sitter's estimator is that in our formula the deviation is around the full sample estimate $\hat{\theta}$ whereas in Shao-Sitter's formula the deviation is around the average of the bootstrap estimates $\bar{\theta}^*$. The balanced repeated replication method (BRR) is implemented in WesVar PC, but the bootstrap method is not. Abramson et al. (1996) suggests that with any BRR software package, the BRR option should be specified for 1993-94 SASS data analysis. The formulae used in WesVar PC for the BRR option is the formula we used above. In general,

$$\frac{1}{B}\sum_{b=1}^{B}\left(\hat{\theta}^{*b}-\bar{\theta}^{*}\right)^2 \le \frac{1}{B}\sum_{i=1}^{B}\left(\hat{\theta}_i-\hat{\theta}\right)^2 = \frac{1}{B}\sum_{b=1}^{B}\left(\hat{\theta}^{*b}-\bar{\theta}^{*}\right)^2 + \left(\bar{\theta}^{*}-\hat{\theta}\right)^2$$

here $\bar{\theta}^{*} = B^{-1}\sum_{b=1}^{B}\hat{\theta}^{*b}$. Notice $E\left(\bar{\theta}^{*}-\hat{\theta}\right)^2 = E_P E_B\left(\bar{\theta}^{*}-\hat{\theta}\right)^2$. Here E_P is with respect to sample design, E_B is with respect to bootstrap subsampling, and typically $E_B\left(\bar{\theta}^{*}\right)=\hat{\theta}$. Therefore $E_B\left(\bar{\theta}^{*}-\hat{\theta}\right)^2 = Var_B\left(\bar{\theta}^{*}\right)$. An unbiased estimator of $Var_B\left(\bar{\theta}^{*}\right)$ is $\hat{V}_B\left(\bar{\theta}^{*}\right)=\frac{1}{B}\frac{1}{B-1}\sum_{b=1}^{B}\left(\hat{\theta}^{*b}-\bar{\theta}^{*}\right)^2$. Therefore

$$\frac{1}{B}\sum_{i=1}^{B}\left(\hat{\theta}_i-\hat{\theta}\right)^2 \approx \left(1+\frac{1}{B-1}\right)\frac{1}{B}\sum_{b=1}^{B}\left(\hat{\theta}^{*b}-\bar{\theta}^{*}\right)^2 .$$

When B is large the bias in variance estimation is small and can be easily corrected by factor $(B-1)/B$. In our study, we compare standard error estimates instead of variance estimates and $B = 48$, so the adjustment factor is $\sqrt{47/48} \approx 0.99$. We do not apply this adjustment because it is close to 1. In addition, we use the same formula to calculate both the standard error estimates cooperating imputation variance and the standard error estimates without cooperating imputation variance. And the ratio of these two types of standard error estimates is used as the measurement of the difference. Therefore, the adjustment factor has no effect on this ratio.

The variables used for this study include 6 categorical variables and 7 continuous variables. Their stage 2 imputation—method (4), rates range from 2 percent to 25 percent (see table 1).

During stage 2 imputation, method (4), a hot deck method, was used to fill items that had missing values. The procedure started with the specification of imputation classes defined by certain relevant variables (matching variables). Then the records were sorted by STGROUP (Groups of states with similar schools) / STATE / TEALEVEL (Instructional level for teacher) / GRADELEV (Grade levels taught this year) / URB (Type of community where school located) / TEAFIELD (Teaching assignment field) / ENROLMNT (Number of students enrolled in the school). The records were then treated sequentially. A nonmissing *y*-variable was used as a starting point for the process. If a record had a response for the *y*-variable, that value replaced the value previously stored for its imputation class. If the record had a missing response, it was assigned the value currently stored for its imputation class. If there was no donor in the class, the class was collapsed with another class. The matching variables and collapse order are listed in table 7 and table 8.

Most of the variables used for sorting or matching the records are not included in the data file; they had to be reconstructed by using other variables in the data file. This caused a discrepancy between the data imputed for this study and the original imputed data in the file. To prevent confounding the imputation difference with imputation variance, we imputed the full sample with our sorting and matching variables and denote this imputed full sample as s_0. This is the sample used in the variance estimation (see imputation procedure step 3 above).

From Table 2 to Table 6, we compare standard errors which do not take the imputation variance into account ($ste\left(\hat{\theta}\right)$) with the standard errors incorporated with imputation variance ($ste_I\left(\hat{\theta}\right)$). It is important to emphasize that both $ste_I\left(\hat{\theta}\right)$ and $ste\left(\hat{\theta}\right)$ are estimates of standard errors instead of true standard errors and therefore both of them are also subjected to sampling errors.

Table 2 compares standard errors for the total estimator of continuous variables. The output shows the imputation does not inflate the variance for the total very much. For variable T0985, the standard error increases only 7 percent even though the imputation rate is as high as 27 percent.

Table 3 compares standard errors for the average per person estimators of continuous variables. The underlying estimator is actually a nonlinear estimator. When the imputation rate is high, inflation to the variance can be very high, too. For example, variable T0985 now shows $ste_I\left(\hat{\theta}\right)$ is 41 percent higher than $ste\left(\hat{\theta}\right)$. So if the imputed data are treated as true values, the underestimation can be severe.

Table 4 compares standard errors for the total estimator of categorical variables. Here the total estimates are estimated total counts in each category. Notice the inflation in variance is larger than the total estimators of continuous variables. This might be due to the fact that the sample sizes of the categorical variables are smaller (there is more legitimate skipping for these items). It also shows that when imputation rates get higher, the increase in standard errors also gets larger although the increase is not exactly linear. Now variable T0040 shows the biggest difference: 2.04.

Table 5 compares standard errors for the percentage estimators of discrete variables. Here the percentage is the estimated percent of units in each category. The underlying estimators are nonlinear estimators. The results are quite similar to those in table 4.

Table 6 compares standard errors for the ratio estimators of continuous variables. Variable BASIC is the ratio of teacher's basic salary to teacher's total income. Variable INSCH is the ratio of teacher's total income at school to teacher's total income. OUTSCH is the ratio of teacher's total income from outside of school to teacher's total income. ADITION is teacher's other income from school (total income inside school minus base salary) to teacher's total income. IN_OUT is teacher's total income inside school to teacher's total income outside school. Although some variables used for the ratios have high imputation rates (T1440, for example, has a 21.3% imputation rate) the increase in standard errors are very small. Again, for continuous variables, we observed smaller inflation in standard error.

4. Summary and Suggestion

The techniques developed so far for the variance estimation of imputed data are not yet easy to implement or operationally convenient. Shao and Sitter's method is appealing but requires repeated imputations, so for large scale surveys the data files become too large.

However, our empirical study shows that using the hot deck imputation method in the 1993-94 SASS can seriously affect the standard error.

But notice that the majority of items have very low stage 2 (hot deck) imputation rates. For the SASS 1993-94 Public School Teacher component, only 11 out of 249 items had stage 2 imputation rates above 10 percent (see Gruber, Rohr, and Fondelier, 1996, figure VIII-24, pp. 231-235). We used six of those items for this study. And, when the imputation rate is low, the inflation in variance is not severe, especially for continuous type variables. We believe it is feasible for NCES to compute the imputation inflation for the total and ratio estimators for the few items that have high imputation rates and document the problem with next user's manual. This will alert secondary users to the possible magnitude of the imputation variance.

Table 1: Variables used in this study

Name	Label	Stage 2 imputation rate (%)	Type
T0030	2 Full/Part-time teacher at this school	11.8	5 Categories
T0035	3A Have other assignment at this sch	9.8	Dichotomous
T0040	3B What is other assignment at this sch	24.0	6 Categories
T0140	11D Consecutive yrs teaching since break	5.2	Continuous
T0435	28A Any mathematics courses taken	5.7	Dichotomous
T0645	32B Programs changed views on teaching	2.0	5 Categories
T0860	40B(4) Number of students in the class	13.6	Continuous
T0985	41C Number of separate classes taught	27.0	Continuous
T1420	53B(1) Academic yr base tchng salary	8.3	Continuous
T1430	53B(2) Additional compensation earned	4.0	Continuous
T1440	53B(3) Earning from job outside sch sys	21.3	Continuous
T1455	53B(5) Income earned from other source	5.9	Continuous
T1520	55 Total income of all HHD family member	25.0	12 Categories

Source: Abramson et al. (1996).

Table 2: Standard error comparison for total estimates of continuous variables

Name	Stage 2 imputation rate (%)	Estimate	$ste(\hat{\theta})$	$ste_I(\hat{\theta})$	$ste_I(\hat{\theta})/ste(\hat{\theta})$
T0140	5.2	8985367	154697	153875	0.99
T0860	13.6	24958128	411554	417361	1.01
T0985	27.0	2107888	72049	77165	1.07
T1420	8.3	86349560396	805679800	808307241	1.00
T1430	4.0	1865774738	36016613	37220591	1.03
T1440	21.3	2179435663	87253029	89579851	1.03
T1455	5.9	588847739	20784683	20928990	1.01

Table 3: Standard error comparison for average estimates of continuous variables

Name	Stage 2 imputation rate (%)	Estimate*	$ste(\hat{\theta})$	$ste_I(\hat{\theta})$	$ste_I(\hat{\theta})/ste(\hat{\theta})$
T0140	5.2	11.01	0.085	0.082	0.96
T0860	13.6	22.79	0.077	0.085	1.10
T0985	27.0	12.79	0.157	0.222	1.41
T1420	8.3	33713.26	88.146	89.404	1.01
T1430	4.0	2093.88	28.232	29.667	1.05
T1440	21.3	4384.44	161.861	170.351	1.05
T1455	5.9	1676.05	48.636	50.182	1.03

* These estimates are average per teacher.

Table 4: Standard error comparison for total estimates of discrete variables

Name	Stage 2 imputation rate (%)	Categories	Estimate	$ste(\hat{\theta})$	$ste_I(\hat{\theta})$	$ste_I(\hat{\theta}) / ste(\hat{\theta})$
T0030	11.8					
		1	12994	1662	1835	1.10
		2	31489	2190	2502	1.14
		3	97607	3719	4156	1.12
		4	52767	2583	2871	1.11
		5	36706	1993	2748	1.38
T0035	9.8					
		1	54006	1969	2130	1.08
		2	166845	4162	4161	1.00
T0040	24.0					
		1	9613	1210	1739	1.44
		2	11737	864	1760	2.04
		3	5093	803	1015	1.26
		4	12311	849	1465	1.73
		5	26962	1844	2335	1.27
		6	5543	715	1158	1.62
T0435	5.7					
		1	2001004	17316	17157	0.99
		2	560289	8838	8807	1.00
T0645	2.0					
		1	122310	4354	4298	0.99
		2	822249	10566	10638	1.01
		3	498908	8204	8187	1.00
		4	711355	10300	10452	1.01
		5	103472	3174	3105	0.98
T1520	25.0					
		1	173	57	82	1.45
		2	863	185	301	1.63
		3	8850	698	723	1.03
		4	72952	2592	3045	1.18
		5	123771	4036	4804	1.19
		6	154036	3771	4152	1.10
		7	174850	4497	5301	1.18
		8	404821	6425	7594	1.18
		9	434259	8408	9091	1.08
		10	523142	8156	10362	1.27
		11	438739	8604	9664	1.12
		12	224836	5327	6480	1.22

Table 5: Standard error comparison for percentage estimates of discrete variables

Name	Stage 2 imputation rate (%)	Categories	Estimate (%)	$ste(\hat{\theta})$	$ste_I(\hat{\theta})$	$ste_I(\hat{\theta}) / ste(\hat{\theta})$
T0030	11.8					
		1	5.61	0.691	0.763	1.10
		2	13.60	0.838	0.991	1.18
		3	42.15	1.383	1.645	1.19
		4	22.79	1.019	1.150	1.13
		5	15.85	0.882	1.195	1.35
T0035	9.8					
		1	24.45	0.775	0.842	1.09
		2	75.55	0.775	0.842	1.09
T0040	24.0					
		1	13.49	1.549	2.392	1.54
		2	16.47	1.169	2.443	2.09
		3	7.15	1.098	1.411	1.29
		4	17.28	1.227	2.038	1.66
		5	37.84	1.861	2.835	1.52
		6	7.78	0.912	1.562	1.71
T0435	5.7					
		1	78.12	0.284	0.279	0.98
		2	21.88	0.284	0.279	0.98
T0645	2.0					
		1	5.42	0.191	0.188	0.98
		2	36.41	0.359	0.364	1.01
		3	22.09	0.283	0.291	1.03
		4	31.50	0.339	0.341	1.01
		5	4.58	0.136	0.132	0.97
T1520	25.0					
		1	0.01	0.002	0.003	1.60
		2	0.03	0.007	0.012	1.68
		3	0.35	0.027	0.028	1.04
		4	2.85	0.099	0.114	1.15
		5	4.83	0.145	0.176	1.22
		6	6.01	0.133	0.149	1.12
		7	6.83	0.172	0.199	1.16
		8	15.81	0.215	0.280	1.30
		9	16.95	0.291	0.332	1.14
		10	20.42	0.292	0.368	1.26
		11	17.13	0.293	0.349	1.19
		12	8.78	0.204	0.248	1.21

Table 6: Standard error comparison for ratio estimates of continuous variables

Basic = T1420/(T1420 + T1430 + T1440 + T1455)
Insch = (T1420 + T1430)/(T1420 + T1430 + T1440 + T1455)
Outsch=T1440/(T1420 + T1430 + T1440 + T1455)
Addition=T1430/(T1420 + T1430 + T1440 + T1455)
In_out=(T1420 + T1430)/(T1440 + T1455)

Name	Stage 2 Imputation rate (%)	Estimate	$ste(\hat{\theta})$	$ste_I(\hat{\theta})$	$ste_I(\hat{\theta}) / ste(\hat{\theta})$
Basic	--	0.94907	0.000966	0.000977	1.01
Insch	--	0.96957	0.000909	0.000938	1.03
Outsch	--	0.02395	0.0008819	0.0009020	1.02
Addition	--	0.02051	0.0003578	0.0003757	1.05
In_out	--	31.87	0.9823	1.010	1.03

Table 7: Public School Teacher (SASS-4A) matching variables

Items	Matching variables
T0030, T0035, T0040	STGROUP, STATE, TEALEVEL, URB, ENR
T0140	STGROUP, STATE, TEALEVEL, AGE, HIGHDEG
T0435, T0645	STGROUP, STATE, TEALEVEL, HIGHDEG, TEAEXPER
T0860	STGROUP, TEALEVEL
T0985	STGROUP, STATE, TEALEVEL, FULPTIME, TEAEXPER
T1420, T1430,T1440, T1455	STGROUP, STATE, TEALEVEL, URB, HIGHDEG, TEAEXPER
T1520	STGROUP, STATE, TEALEVEL, URB, HIGHDEG, TEAEXPER

Source: Gruber, Rohr, and Fondelier (1996), figure VIII-28.

Table 8: Public School Teacher (SASS-4A) order of collapse

Items	Order of collapse
T0030, T0035, T0040	ENR, URB, STATE
T0140	HIGHDEG, AGE, STATE
T0435, T0645	TEAEXPER, HIGHDEG, STATE
T0860	TEALEVEL
T0985	TEAEXPER, FULPTIME, STATE
T1420, T1430,T1440, T1455	TEAEXPER, HIGHDEG, STATE
T1520	TEAEXPER, HIGHDEG, TEALEVEL

Source: Gruber et al. (1996), figure VIII-28.

Appendix

This appendix presents results we derived for Kaufman's method. In Kaufman's method, a residual is defined for each $k \in s$:

$$\tilde{d}_k^R = \begin{cases} 0 & \text{if } k \in r \\ (2I_k - 1)(y_{j_k 2} - y_{j_k 1}) & \text{if } k \in nr \end{cases}$$

where j_k is a missing item within missing item *k*'s imputation cell and $(y_{j_k 2} - y_{j_k 1})$ is the difference between the two nearest neighbors (donors) of j_k. Missing item j_k is selected independently from *k*'s imputation cell with known selection probability; for example, with selection probability proportional to design weights w_k. Then the residual is attached to $\tilde{y}_k$ to form another quantity $\hat{Y}$ used for the purpose of variance estimation:

$$\hat{Y} = \hat{y}_\bullet + \hat{d}^R = \sum_{k \in s} w_k \left(\tilde{y}_k + \tilde{d}_k^R\right).$$

The variance of $\hat{Y}$ contains variability from both $\hat{y}_\bullet$ and $\hat{d}^R$.

Lemma 1. Let $\tilde{y}_\bullet = \sum_{k \in s} w_k \tilde{y}_k$ and $\bar{y}_\bullet = 1/2\left(\hat{y}_1 + \hat{y}_2\right)$. Here $\hat{y}_1 = \sum_{k \in r} w_k y_k + \sum_{k \in nr} w_k y_{k1}$, $\hat{y}_2 = \sum_{k \in r} w_k y_k + \sum_{k \in nr} w_k y_{k2}$, and E_2 is with respect to the imputation selection and the residual selection. Then $E_2(\hat{y}_\bullet) = \bar{y}_\bullet$.

Proof: $E_2(\hat{y}_\bullet) = E_2\left(\sum_{k \in s} w_k \tilde{y}_k\right) = \sum_{k \in r} w_k y_k + \sum_{k \in nr} w_k E_2\left[y_{k1} I_k + y_{k2}(1 - I_k)\right]$

$$= \sum_{k \in r} w_k y_k + \sum_{k \in nr} w_k \left(0.5 y_{k1} + 0.5 y_{k2}\right) = 1/2\left(\hat{y}_1 + \hat{y}_2\right)$$

$$= \bar{y}_\bullet.$$

Lemma 2. Let $\hat{Y} = \hat{y}_\bullet + \hat{d}^R$. Here $\hat{d}^R = \sum_{k \in s} w_k \hat{d}_k^R$ and

$$\tilde{d}_k^R = \begin{cases} 0 & \text{if } k \in r \\ (2I_k - 1)(y_{j_k 2} - y_{j_k 1}) & \text{if } k \in nr. \end{cases}$$

Then $E_2\left(\hat{Y}\right) = \bar{y}_\bullet$.

Proof: Since $E_2\left(\hat{Y}\right) = E_2(\hat{y}_\bullet) + E_2\left(\hat{d}^R\right)$, we only need to show $E_2\left(\hat{d}^R\right) = 0$.

Actually

$$E_2\left(\hat{d}^R\right) = \sum_{k \in nr} w_k E_2\left[(2I_k - 1)(y_{j_k 2} - y_{j_k 1})\right] = 0.$$

Combine Lemma 1 and Lemma 2: we can see $E_2\left(\hat{Y}\right) = E_2(\hat{y}_\bullet)$.

Lemma 3. $V_1 E_2\left(\hat{Y}\right) = V_1(\bar{y}_\bullet - \hat{y}) + V_1(\hat{y}) + 2COV_1(\bar{y}_\bullet - \hat{y}, \hat{y})$. Here V_1 is with respect to the sample design and the nonresponse mechanism.

Proof: $V_1 E_2\left(\hat{Y}\right) = V_1 E_2(\hat{y}_\bullet)$ Lemma 2

$= V_1(\bar{y}_\bullet)$ Lemma 1

$= V_1(\bar{y}_\bullet - \hat{y} + \hat{y})$

$$= V_1(\bar{y}_\bullet - \hat{y}) + V_1(\hat{y}) + 2COV_1(\bar{y}_\bullet - \hat{y}, \hat{y}).$$

Lemma 4. $\hat{y}_\bullet$ and $\hat{d}^R$ are uncorrelated with respect to imputation selection and residual selection; that is, $V_2\left(\hat{y}_\bullet + \hat{d}^R\right) = V_2(\hat{y}_\bullet) + V_2\left(\hat{d}^R\right)$.

Proof: Notice that

$$V_2\left(\hat{y}_\bullet + \hat{d}^R\right) = E_I V_R\left(\hat{y}_\bullet + \hat{d}^R\right) + V_I E_R\left(\hat{y}_\bullet + \hat{d}^R\right) = E_I V_R\left(\hat{d}^R\right) + V_I\left[E_R(\hat{y}_\bullet) + E_R\left(\hat{d}^R\right)\right]$$

$$= E_I V_R\left(\hat{d}^R\right) + V_I\left[\hat{y}_\bullet + E_R\left(\hat{d}^R\right)\right]$$

$$= E_I V_R\left(\hat{d}^R\right) + V_I(\hat{y}_\bullet) + V_I E_R\left(\hat{d}^R\right) + 2COV_I\left(\hat{y}_\bullet, E_R\left(\hat{d}^R\right)\right)$$

and

$$V_2(\hat{y}_\bullet) + V_2\left(\hat{d}^R\right) = E_I V_R(\hat{y}_\bullet) + V_I E_R(\hat{y}_\bullet) + E_I V_R\left(\hat{d}^R\right) + V_I E_R\left(\hat{d}^R\right)$$

$$= V_I(\hat{y}_\bullet) + E_I V_R\left(\hat{d}^R\right) + V_I E_R\left(\hat{d}^R\right).$$

Therefore, we only need to show $2COV_I\left(\hat{y}_\bullet, E_R\left(\hat{d}^R\right)\right) = 0$. Notice

$$COV_I\left(\hat{y}_\bullet, E_R\left(\hat{d}^R\right)\right) = E_I\left[\hat{y}_\bullet E_R\left(\hat{d}^R\right)\right] - E_I(\hat{y}_\bullet) E_I E_R\left(\hat{d}^R\right),$$

and

$$E_R\left(\hat{d}^R\right) = \sum_{k\in s} w_k E_R\left(\tilde{d}_k^R\right) = \sum_{k\in nr} w_k (2I_k - 1) E_R\left(y_{j_k 2} - y_{j_k 1}\right) = \sum_{k\in nr} w_k (2I_j - 1)\mu_k^R .$$

Here $\mu_k^R = E_R\left(y_{j_k 2} - y_{j_k 1}\right)$. Also notice

$$E_I\left[\hat{y}_\bullet E_R\left(\hat{d}^R\right)\right] = E_I\left\{\left[\sum_{k\in r} w_k y_k + \sum_{k\in nr} w_k\left(y_{k1} I_k + y_{k2}(1 - I_k)\right)\right]\left[\sum_{h\in nr} w_h (2I_h - 1)\mu_h^R\right]\right\}$$

$$= E_I\left[\sum_{k\in r}\sum_{h\in nr} w_k y_k w_h (2I_h - 1)\mu_h^R + \sum_{k\in nr}\sum_{h\in nr} w_k\left(y_{k1} I_k + y_{k2}(1 - I_k)\right) w_h (2I_h - 1)\mu_h^R\right]$$

$$= 0 + \sum_{k\in nr}\sum_{h\in nr} w_k w_h \mu_h^R\left[y_{k1} E_I(2I_k I_h - I_k) + y_{k2} E_I(1 - I_k)(2I_h - 1)\right]$$

$$= 0,$$

and

$$E_I E_R\left(\hat{d}^R\right) = E_I\left[\sum_{k\in nr} w_k (2I_k - 1)\mu_k^R\right] = 0;$$

therefore,

$$COV_I\left(\hat{y}_\bullet, E_R\left(\hat{d}^R\right)\right) = 0.$$

Lemma 5. $V(\hat{y}) + V(\hat{y} - \hat{y}_\bullet) = V_1(\hat{y}) + V_1(\hat{y} - \bar{y}_\bullet) + E_1 V_2(\hat{y}_\bullet)$. Here subscript 1 is with respect to the sampling design and nonresponse mechanism, subscript 2 is with respect to the imputation selection and the residual selection. No subscript is with respect to all variance components.

Proof: $V(\hat{y}) = V_1 E_2(\hat{y}) + E_1 V_2(\hat{y}) = V_1(\hat{y})$,

$$V(\hat{y} - \hat{y}_\bullet) = V_1 E_2(\hat{y} - \hat{y}_\bullet) + E_1 V_2(\hat{y} - \hat{y}_\bullet)$$

$$= V_1(\hat{y} - \bar{y}_\bullet) + E_1 V_2(\hat{y}_\bullet) \qquad \text{Lemma 1}$$

Theorem 1. $V\left(\hat{Y}\right)=V\left(\hat{y}\right)+V\left(\hat{y}-\hat{y}_{\bullet}\right)+2COV_1\left(\bar{y}_{\bullet}-\hat{y},\hat{y}\right)+E_1V_2\left(\hat{d}^R\right)$.

Proof: $V\left(\hat{Y}\right)=V_1E_2\left(\hat{Y}\right)+E_1V_2\left(\hat{Y}\right)$

$$=V_1\left(\bar{y}_{\bullet}-\hat{y}\right)+V_1\left(\hat{y}\right)+2COV_1\left(\bar{y}_{\bullet}-\hat{y},\hat{y}\right)+E_1V_2\left(\hat{y}_{\bullet}+\hat{d}^R\right) \qquad \text{Lemma 3}$$

$$=V_1\left(\bar{y}_{\bullet}-\hat{y}\right)+V_1\left(\hat{y}\right)+2COV_1\left(\bar{y}_{\bullet}-\hat{y},\hat{y}\right)+E_1V_2\left(\hat{y}_{\bullet}\right)+E_1V_2\left(\hat{d}^R\right) \qquad \text{Lemma 4}$$

$$=V\left(\bar{y}_{\bullet}-\hat{y}\right)+V\left(\hat{y}\right)+2COV_1\left(\bar{y}_{\bullet}-\hat{y},\hat{y}\right)+E_1V_2\left(\hat{d}^R\right). \qquad \text{Lemma 5}$$

Theorem 2. $V\left(\hat{Y}\right)=V\left(\hat{y}_{\bullet}\right)+E_1V_2\left(\hat{d}^R\right)$.

Proof: $V\left(\hat{Y}\right)=V_1E_2\left(\hat{Y}\right)+E_1V_2\left(\hat{Y}\right)$

$$=V_1E_2\left(\hat{y}_{\bullet}\right)+E_1V_2\left(\hat{y}_{\bullet}\right)+E_1V_2\left(\hat{d}^R\right) \qquad \text{Lemma 1, 2, and 4}$$

$$=V\left(\hat{y}_{\bullet}\right)+E_1V_2\left(\hat{d}^R\right).$$

Theorem 3. $V\left(\hat{y}_{\bullet}\right)=V\left(\hat{y}\right)+V\left(\hat{y}-\hat{y}_{\bullet}\right)+2COV_1\left(\bar{y}_{\bullet}-\hat{y},\hat{y}\right)$

Proof follows directly from theorem 1 and theorem 2.
The result in theorem 3 is actually the same as formula (4) of section 2.2, as it should be.

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