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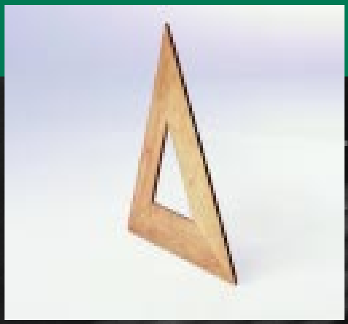
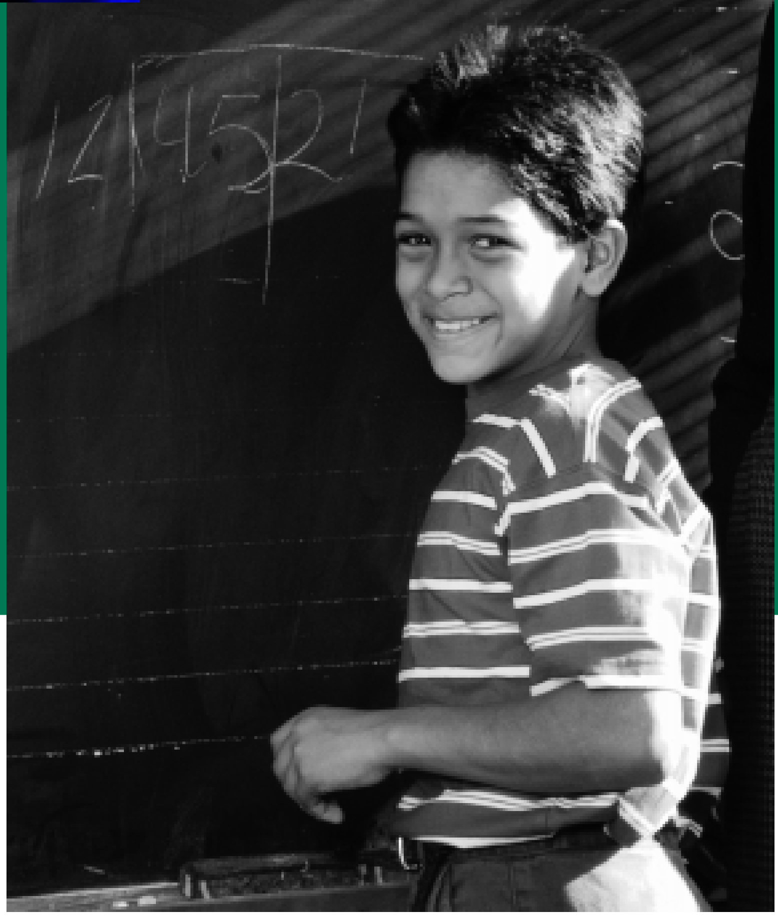
STUDENT WORK & TEACHER PRACTICES IN MATHEMATICS





THE NATION'S REPORT CARD naep

U.S. Department of Education
Office of Educational Research and Improvement

NCES 1999-453

What is The Nation's Report Card?

THE NATION'S REPORT CARD, the National Assessment of Educational Progress (NAEP), is the only nationally representative and continuing assessment of what America's students know and can do in various subject areas. Since 1969, assessments have been conducted periodically in reading, mathematics, science, writing, history, geography, and other fields. By making objective information on student performance available to policymakers at the national, state, and local levels, NAEP is an integral part of our nation's evaluation of the condition and progress of education. Only information related to academic achievement is collected under this program. NAEP guarantees the privacy of individual students and their families.

NAEP is a congressionally mandated project of the National Center for Education Statistics, the U.S. Department of Education. The Commissioner of Education Statistics is responsible, by law, for carrying out the NAEP project through competitive awards to qualified organizations. NAEP reports directly to the Commissioner, who is also responsible for providing continuing reviews, including validation studies and solicitation of public comment, on NAEP's conduct and usefulness.

In 1988, Congress established the National Assessment Governing Board (NAGB) to formulate policy guidelines for NAEP. The Board is responsible for selecting the subject areas to be assessed from among those included in the National Education Goals; for setting appropriate student performance levels; for developing assessment objectives and test specifications through a national consensus approach; for designing the assessment methodology; for developing guidelines for reporting and disseminating NAEP results; for developing standards and procedures for interstate, regional, and national comparisons; for determining the appropriateness of test items and ensuring they are free from bias; and for taking actions to improve the form and use of the National Assessment.

The National Assessment Governing Board

NATIONAL CENTER FOR EDUCATION STATISTICS

Student Work and Teacher Practices in Mathematics

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March 1999

U.S. Department of Education
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March 1999

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Chapter 1

Introduction

The National Assessment of Educational Progress (NAEP) is mandated by the United States Congress to survey the educational accomplishments of U.S. students and monitor changes in those accomplishments. For more than 25 years, NAEP has assessed the educational achievement of fourth-, eighth-, and twelfth-grade students in selected subject areas, making it the only nationally representative and continuing assessment of what U.S. students know and can do. NAEP assessments are based on content frameworks and specifications developed through a national consensus process involving teachers, curriculum experts, parents, and members of the general public. The frameworks are designed to reflect a balance among the emphases suggested by current instructional efforts, curriculum reform, contemporary research, and desirable levels of achievement.

Purpose and Audience for the Report

In 1996, NAEP assessed the abilities of students at grades 4, 8, and 12 in the subjects of mathematics and science. The first release of results from the mathematics assessment appeared in the *NAEP 1996 Mathematics Report Card*,[1] a report designed to provide policy makers and the public with a broad view of student achievement.

The current report, which provides a more detailed perspective on mathematics achievement and practices in 1996, is primarily for teachers, curriculum specialists, and school administrators. To illustrate what students know and can do, the report presents examples of student work in five different content strands of mathematics. Information on current instruction in mathematics classes, as reported by students and teachers, also is included.

A companion report, *School Policies and Practices Affecting Instruction in Mathematics*,[2] provides information on school policies and other practices affecting mathematics education.

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[1] Reese, C. M., Miller, K. E., Mazzeo, J., & Dossey, J. A. (1997). *NAEP 1996 mathematics report card for the nation and the states.* Washington, DC: National Center for Education Statistics.

[2] Hawkins, E. F., Stancavage, F., & Dossey, J. A. (1998). *School policies and practices affecting instruction in mathematics.* Washington, DC: National Center for Education Statistics.

1996 Mathematics Framework

The design of the NAEP 1996 mathematics assessment was guided by a framework that was closely aligned with the frameworks used in 1990 and 1992.[3] This framework was influenced by the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics*[4] and was updated prior to use in 1996 to better reflect contemporary curricular emphases and objectives. However, a connection with the 1990 and 1992 assessments was maintained in order to measure trends in student performance.

Content strands

The framework for the 1996 mathematics assessment included a broad content domain consisting of five content strands: Number Sense, Properties, and Operations; Measurement; Geometry and Spatial Sense; Data Analysis, Statistics, and Probability; and Algebra and Functions. For descriptions of the content covered in these strands, see Chapters 3–7, which describe student performance in each content strand. Table 1.1 presents the percentage distribution of questions prescribed by the framework across the content strands for each grade level and shows the changes from 1990 and 1992 to 1996. Separate subscales were produced for the five content strands that summarize the results for each strand. Questions that tap content from more than one strand were grouped according to their primary content classification.

Table 1.1 ***Percentage Distribution of Questions by Content Strand and Grade***

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	Grade 4			Grade 8			Grade 12		
	1990	1992	1996	1990	1992	1996	1990	1992	1996
Content Area									
Number Sense, Properties, & Operations[a]	45	45	40	30	30	25	25	25	20
Measurement	20	20	20	15	15	15	15	15	15
Geometry & Spatial Sense[b]	15	15	15	20	20	20	20	20	20
Data Analysis, Statistics, & Probability	10	10	10	15	15	15	15	15	20
Algebra & Functions	10	10	15	20	20	25	25	25	25

[a] Approximately half the questions in 1996 at each grade level involved some aspect of estimation.
[b] At grade 12 in 1996, approximately 25 percent of the geometry questions involved topics in coordinate geometry.
SOURCE: *NAEP 1996 Mathematics Report Card for the Nation and the States.*

[3] National Assessment Governing Board (1996). *Mathematics framework for the 1996 National Assessment of Educational Progress*. Washington, DC: Author.

[4] National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

Mathematical abilities

A domain of general abilities associated with doing mathematics also was included in the framework. These mathematical abilities — conceptual understanding, procedural knowledge, and problem solving — describe the nature of the knowledge or processes involved in successfully completing the types of mathematical tasks that students are expected to master. For example, conceptual understanding can be viewed as a student's knowing "about" something, while procedural knowledge can be viewed as a student's knowing "how to do" something. These two abilities combined provide a base for problem solving, that is, for recognizing and understanding a problem, formulating a plan or strategy, arriving at a solution, and reflecting upon or evaluating the solution.

Mathematical power

The third domain included in the framework is mathematical power. Mathematical power is defined as a student's ability to reason in mathematical situations; to communicate perceptions and conclusions drawn from a mathematical context; and to connect the mathematical nature of a situation with related mathematical knowledge and information gained from other disciplines or through observation. The cognitive skills of reasoning, communicating, and connecting lie at the foundation of each of the content strands and each of the mathematical abilities.

Assessment questions were classified according to mathematical ability and mathematical power, as well as content. Figure 1.1 shows how the content strands, mathematical abilities, and mathematical power combine to form the framework for the NAEP 1996 mathematics assessment.

Figure 1.1 ***Mathematics Framework for the 1996 Assessment***

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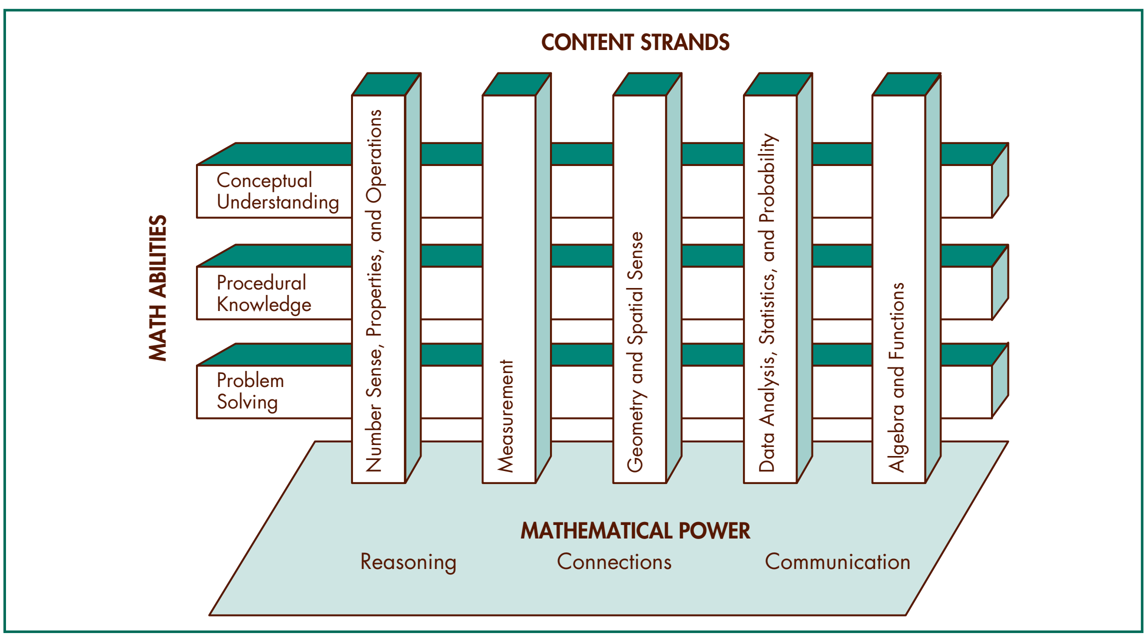

SOURCE: National Assessment Governing Board, *Mathematics Framework for the 1996 National Assessment of Educational Progress.*

Question types

The NAEP mathematics framework also prescribed a mix of question types: multiple-choice, short constructed-response, and extended constructed-response. Multiple-choice questions require students to select the answer that best expresses what they believe is correct. Short constructed-response questions require students to provide a brief response, which might be a numerical result, the correct name or classification for a group of mathematical objects, a drawn example of a given concept, or perhaps a brief written explanation for a given result. Extended constructed-response questions require students to consider a situation that demands more than a numerical response or a short written explanation. The response mode requires that students provide evidence of their work on some aspect of the solution and communicate their decision-making steps in the context of the problem.

Table 1.2 shows the distribution of questions by type for the 1990, 1992, and 1996 assessments. As the table shows, the 1996 assessment continued a shift begun in 1992 toward the use of more constructed-response questions. Current recommendations call for the use of constructed-response questions as a way to assess students' abilities to reason and to communicate mathematically. They provide an added dimension to the information that can be gleaned from multiple-choice questions.

The framework also called for the assessment to incorporate the use of calculators, rulers, protractors (grades 8 and 12 only), and manipulatives (including geometric shapes, three-dimensional models, and spinners).

Table 1.2 ***Distribution of Questions by Type***

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Question Type	Grade 4			Grade 8			Grade 12		
	1990	1992	1996	1990	1992	1996	1990	1992	1996
Multiple-Choice	102	99	81	149	118	102	156	115	99
Short Constructed-Response[a]	41	59	64	42	65	69	47	64	74
Extended Constructed-Response[b]	–	5	13	–	6	12	–	6	11
Total	143	163	158	191	189	183	203	185	184

[a] Short constructed-response questions previously used in the 1990 and 1992 assessments were scored dichotomously (right/wrong). New short constructed-response questions included in the 1996 assessment were scored to allow for partial credit.

[b] No extended constructed-response questions were included in the 1990 assessment.

SOURCE: *NAEP 1996 Mathematics Report Card for the Nation and the States*.

Estimating Mathematics Achievement

Information from both the multiple-choice and constructed-response questions was combined in order to estimate mathematics achievement in the five content strands and overall. Constructed-response questions were first scored by trained readers using criteria that distinguished among two to five levels of performance.[5] When the questions were anchored to the NAEP scale and used in the estimation of students' mathematics achievement, each of the scoring levels was anchored separately. However, for a few of the questions, adjacent score categories were collapsed because the responses lacked sufficient structure to maintain statistically the distinctions implied by the hand scoring. These instances will be noted in the text.

In addition, because of the broad content domain covered by the assessment and the need to reduce the burden on individual schools and students, no student who participated in the NAEP mathematics assessment answered all of the questions. Rather, each student was administered a portion of the assessment, and then data across students were combined to provide estimates of the achievement of fourth-, eighth-, and twelfth-grade students overall and within important subgroups, such as those defined by gender or race/ethnicity. No individual student scores were derived. Further details on scoring and other technical aspects of the assessment are provided in Appendix A.

[5] Each NAEP assessment contains questions that were used before (for trend analysis), as well as new questions. Short constructed-response questions that had previously appeared in the 1990 and 1992 assessments were scored right or wrong in 1996, as they had been in the earlier assessments. All other constructed-response questions were scored using more complex, partial-credit guidelines.

Reporting NAEP Results

Student performance on NAEP assessments has been reported using a variety of measures. Results for the main NAEP mathematics assessment are reported using the NAEP composite mathematics scale, which summarizes performance across five separate subscales — one for each of the five content strands. Achievement levels, authorized by the NAEP legislation and adopted by the National Assessment Governing Board (NAGB), help to make these scaled results meaningful and interpretable. The achievement levels are defined by broadly representative panels of teachers, education specialists, and members of the general public, and they therefore represent collective judgments about what students should know and be able to do relative to the content reflected in the NAEP frameworks. Brief policy descriptions of the levels are provided in Figure 1.2.[6]

Figure 1.2 ***Policy Definitions of NAEP Achievement Levels***

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Basic	This level denotes partial mastery of prerequisite knowledge and skills that are fundamental for proficient work at each grade.
Proficient	This level represents solid academic performance for each grade assessed. Students reaching this level have demonstrated competence in challenging subject matter, including subject-matter knowledge, application of such knowledge to real-world situations, and analytical skills appropriate to the subject matter.
Advanced	This level signifies superior performance.

SOURCE: *NAEP 1996 Mathematics Report Card for the Nation and the States.*

It should be noted that setting achievement levels is a relatively new process for NAEP, and it is still in transition. Some evaluations have concluded that the percentage of students at certain levels may be underestimated.[7] On the other hand, critiques of those evaluations have asserted that the weight of the empirical evidence does not support such conclusions.[8] A further review is currently being conducted by the National Academy of Sciences.

[6] Further information about NAEP scale construction and about the achievement levels can be found in Reese, C. M., Miller, K. E., Mazzeo, J., & Dossey, J. A. (1997). op. cit.

[7] United States General Accounting Office Report to Congressional Requestors (1993). *Education achievement standards: NAGB's approach yields misleading interpretations.* Washington, DC: United States General Accounting Office.

[8] Cizek, G. (1993). *Reactions to National Academy of Education report*. Washington, DC: National Assessment Governing Board.

The student achievement levels in this report have been developed carefully and responsibly, and the procedures used have been refined and revised as new technologies have become available. Upon review of the available information, the Commissioner of Education Statistics has judged that the achievement levels are in a developmental status. However, the Commissioner and the Governing Board also believe that the achievement levels are useful and valuable for reporting on the educational achievement of students in the United States.[9]

Organization of the Report

The body of this report is divided into two main sections: Section I — a report on student work, and Section II — a report on classroom instruction. Some readers may prefer to read Section I before reading Section II; however, others may prefer to read about classroom instruction (Section II) before reading about student performance (Section I).

Section I (Chapters 2–7) provides information on trends in achievement since 1990, as well as examples of student performance in each of the five mathematics content strands. Chapter 2 presents summaries of performance for 1990, 1992, and 1996. Results for each of the five content strands are presented by average scale score for all students and separately for some of the more common demographic and education groupings, such as gender, race/ethnicity,[10] and, at grades 8 and 12, the kinds of courses taken. Chapters 3–7 each consider one content strand. Each begins with a brief discussion of the expected knowledge and skills that students are asked to demonstrate in that content strand. Each chapter then presents an item map (a visual representation of the NAEP mathematics scale) for the content strand, with selected questions from the content strand mapped onto the 0 to 500 scale. Finally, sample questions from different points on the map are presented, along with a discussion of student performance on these questions. For constructed-response questions, actual student responses are included to provide the reader with illustrations of partial- and full-credit responses.

Section II includes Chapters 8–10. Chapter 8 describes the mathematics course-taking patterns of eighth- and twelfth-grade students. Chapter 9 discusses classroom activities, including instructional emphases and approaches, assessment activities, and calculator use. Chapter 10 reports on student attitudes about mathematics.

Finally, Chapter 11 presents an overall summary of the report. The chapter summarizes what students know and can do in the five content strands of the NAEP 1996 mathematics assessment, course-taking patterns and classroom practices in mathematics, and student attitudes toward mathematics.

The report also contains two appendices that support the results presented. Appendix A contains an overview of the procedures used for the NAEP 1996 mathematics assessment. Appendix B presents standard errors for the performance data presented in the body of the report.

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[9] For fourth-grade students, 0–213 is defined by the National Assessment Governing Board as below *Basic*, 214–248 is *Basic*, 249–281 is *Proficient*, and 282–500 is *Advanced*; for eighth-grade students, 0–261 is below *Basic*, 262–298 is *Basic*, 299–332 is *Proficient*, and 333–500 is *Advanced*; and for twelfth-grade students, 0–287 is below *Basic*, 288–335 is *Basic*, 336–366 is *Proficient* and 367–500 is *Advanced*.

[10] In designations of race/ethnicity, White is defined as White non-Hispanic, and Black is defined as Black non-Hispanic. See Appendix A for more detail.

Chapter 2

General Results — Summaries of Performance in Mathematics Content Strands

In this chapter, student performance is examined as it relates to proficiency in the five content strands of the NAEP 1996 mathematics assessment. Summaries of overall performance, as well as performance in the five mathematics content strands, are presented for 1996, 1992, and 1990. Results are presented by average scale score and are shown for demographic and education groupings, such as gender, race/ethnicity, and, for grades 8 and 12, by the types of mathematics courses taken. The five content strands are Number Sense, Properties, and Operations; Measurement; Geometry and Spatial Sense; Data Analysis, Statistics, and Probability; and Algebra and Functions. A brief description of each of the five strands is given in Figure 2.1.

Figure 2.1 *Descriptions of the Five NAEP Mathematics Content Strands*

Number Sense, Properties, and Operations
This content strand focuses on students' understanding of numbers (whole numbers, fractions, decimals, integers, real numbers, and complex numbers), operations, and estimation, and their application to real-world situations. At grade 4, this strand emphasizes the development of number sense through connecting various models to their numerical representations and an understanding of the meaning of addition, subtraction, multiplication, and division. At grade 8, number sense is extended to include positive and negative numbers, and the strand addresses properties and operations involving whole numbers, fractions, decimals, integers, and rational numbers. At grade 12, this strand includes real and complex numbers and allows students to demonstrate competency up to the pre-calculus or calculus level.
Measurement
This content strand focuses on an understanding of the process of measurement and the use of numbers and measures to describe and compare mathematical and real-world objects. Students are asked to identify attributes, select appropriate units and tools, apply measurement concepts, and communicate measurement-related ideas. At grade 4, the strand focuses on time, money, temperature, length, perimeter, area, capacity, weight/mass, and angle measure. At grades 8 and 12, the strand includes these measurement concepts, but the focus shifts to more complex measurement problems that involve volume or surface area or that require students to combine shapes and to translate and apply measures. Eighth- and twelfth-grade students also solve problems involving proportional thinking (such as scale drawing or map reading) and do applications that involve the use of complex measurement formulas.
Geometry and Spatial Sense
This content strand is designed to extend beyond low-level identification of geometric shapes to include transformations and combinations of those shapes. Informal constructions and demonstrations (including drawing representations), along with their justifications, take precedence over more traditional types of compass-and-straightedge constructions and proofs. At grade 4, students are asked to model properties of shapes under simple combinations and transformations, and they are asked to use mathematical communication skills to draw figures from verbal descriptions. At grade 8, students are asked to expand their understanding to include properties of angles and polygons. They are also asked to apply reasoning skills to make and validate conjectures about transformations and combinations of shapes. At grade 12, students are asked to demonstrate an understanding of transformational geometry and to apply concepts of proportional thinking to various geometric situations.

Figure 2.1 (cont)

Descriptions of the Five NAEP Mathematics Content Strands

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Data Analysis, Statistics, and Probability

This content strand emphasizes the appropriate methods for gathering data, the visual exploration of data, various ways of representing data, and the development and evaluation of arguments based on data analysis. At grade 4, students are asked to apply their understanding of numbers and quantities by solving problems that involve data. Fourth graders are asked to interact with a variety of graphs, to make predictions from data and explain their reasoning, to deal informally with measures of central tendency, and to use the basic concepts of chance in meaningful contexts. At grade 8, students are asked to analyze statistical claims and to design experiments, and they are asked to use simulations to model real-world situations. This strand focuses on eighth graders' basic understanding of sampling, their ability to make predictions based on experiments or data, and their ability to use some formal terminology related to probability, data analysis, and statistics. At grade 12, the strand focuses on the ability to apply the concepts of probability and to use formulas and more formal terminology to describe a variety of situations. For twelfth graders, the strand also emphasizes a basic understanding of how to use mathematical equations and graphs to interpret data.

Algebra and Functions

This content strand extends from work with simple patterns at grade 4 to basic algebra concepts at grade 8 to sophisticated analysis at grade 12. It involves not only algebra, but also pre-calculus and some topics from discrete mathematics. Students are expected to use algebraic notation and thinking in meaningful contexts to solve mathematical and real-world problems, specifically addressing an increasing understanding of the use of functions (including algebraic and geometric) as representational tools. The grade 4 assessment involves informal demonstration of students' abilities to generalize from patterns, including the justification of their generalizations. Students are expected to translate between mathematical representations, to use simple equations, and to do basic graphing. At grade 8, the assessment includes more algebraic notation, stressing the meaning of variables and an informal understanding of the use of symbolic representations in problem-solving contexts. Students are asked to use variables to represent a rule underlying a pattern. Eighth graders are asked to demonstrate a beginning understanding of equations and functions and the ability to solve simple equations and inequalities. By grade 12, students are asked about basic algebraic notation and terminology as they relate to representations of mathematical and real-world situations. Twelfth graders are asked to use functions as a way of representing and describing relationships.

SOURCE: *NAEP 1996 Mathematics Report Card for the Nation and the States.*

Interpretation of the Data

In general, mathematics results for all content strands and all groups of students have been improving since 1990. However, not all of these changes were statistically significant, where "statistical significance" means that there is a high level of certainty that the results would not have occurred by chance. Factors that affect statistical significance include the magnitude of the difference between the group averages (e.g., between average performance in 1992 and average performance in 1996), the amount of variability of performance *within* each group, and even the size of the groups surveyed. Thus, for example, improved performance by a specific amount in one of the content strands might be found to be statistically significant for White students, but improvement by the same amount in that strand might not be statistically significant for students of other racial/ethnic backgrounds. Statistically significant differences are noted in the figures and text that follow. All comparisons discussed in this report are statistically significant unless otherwise noted. It is important not to focus on apparent differences that are not statistically significant because these differences might be a result of sampling error. In some cases where differences among groups appear large, but, in fact, are not significant, it is noted in the text that the group differences are not "statistically significant," or there were no "significant differences."[1]

Figure 2.2 presents information on the average proficiency in each content strand for all students in grades 4, 8, and 12 for 1996, 1992, and 1990. The average proficiency on the NAEP composite mathematics scale also is shown. Table 2.1 disaggregates this information by gender, and Figures 2.3–2.8 break it out by race/ethnicity. Average proficiencies for eighth-grade Asian/Pacific Islander students are not included in the figures, however, due to concerns regarding the quality and credibility of the results obtained for this group. Data from the NAEP state assessment program in mathematics also conducted in 1996 provided an independent data source to aid in evaluating the accuracy of the national grade 8 NAEP results for Asian/Pacific Islander students as well as for other subgroups. These results suggested that the 1996 national results may substantially underestimate the actual achievement of the Asian/Pacific Islander group. In view of the potential to misinform, it was decided to omit the national grade 8 Asian/Pacific Islander results from the body of the report.[2] Appendix A includes average scale scores on the national assessment for this group along with a description of the findings that led to this decision.

A brief discussion of observed trends and significant results follows. The discussion refers to Figures 2.2–2.8 and Table 2.1.

[1] See Appendix A Guidelines for Analysis and Reporting for further discussion of determining statistical significance.

[2] Asian/Pacific Islander students are included, however, in performance data for all students.

Trends

Comparisons with 1990

In 1992, significant gains over performance in 1990 were observed in mathematics performance on the composite scale and in each content strand for the general population at all three grade levels. Considerable gains also were evident in most cases for the female, male, and White subgroups of students. The exceptions were 1) males in the Data Analysis, Statistics, and Probability strand at all three grades and in both the Geometry and Spatial Sense content strand and the Measurement content strand at grade 8; and 2) White students in Data Analysis, Statistics, and Probability at grade 12.

Other improvements from 1990 to 1992 were observed for Black students in overall mathematics performance at grade 12, as well as in Geometry and Spatial Sense at grades 4 and 12. Hispanic students improved between 1990 and 1992 in Geometry and Spatial Sense at grades 4 and 12.

The same trends were noted when comparing 1996 performance with 1990 performance. Additionally, males and White students showed improvement in the areas noted as exceptions for 1992. In 1996, Black students showed additional gains in performance relative to 1990 in overall mathematics performance at grade 4, as well as in Number Sense, Properties, and Operations at grade 4 and in Measurement and in Algebra and Functions at grade 12. Hispanic students showed additional gains in Algebra and Functions at all grades, as well as in Geometry and Spatial Sense at grade 4.

Comparisons with 1992

In 1996, improvement over 1992 performance was noted in overall mathematics performance for the general population, males, females, and White students at all grades, with the exception of eighth-grade White students and eighth-grade male students. Improvement also was noted for the general population in Geometry and Spatial Sense and in Algebra and Functions at all grades; and in Number Sense, Properties, and Operations and in Data Analysis, Statistics, and Probability at fourth grade; as well as in Measurement and in Data Analysis, Statistics, and Probability at twelfth grade. Thus, there appears to be continued improvement to 1996 in the content strands of Geometry and Spatial Sense and Algebra and Functions. This is less true for the other content strands, where improvement for at least some grades appears to have leveled out after 1992.

Trends for male, female, and White students were similar to those observed for all students, with the following exceptions: 1) males did not show significant improvement in Geometry and Spatial Sense at fourth grade or in Algebra and Functions at twelfth grade, and 2) females did not show significant improvement in Data Analysis, Statistics, and Probability at fourth grade but did show improvement in this content strand at eighth grade. Additionally, Black and Hispanic students showed improvement in 1996 in Algebra and Functions relative to their performance in 1992 in grades 4 and 8. The same was true for the performance of American Indian students in grade 8.

Subgroups

Gender

As can be seen in Table 2.1, in 1996, gender differences in performance favoring males were observed for grade 4 overall proficiency and for three content strands: Number Sense, Properties, and Operations; Measurement; and Algebra and Functions. At grade 12, gender differences, also favoring males, were observed for two content strands: Measurement, and Geometry and Spatial Sense. There were no significant differences in performance between males and females at grade 8.

Race/ethnicity

Figures 2.3–2.8 show that in 1996, White and Asian/Pacific Islander students at grades 4 and 12 performed better than other ethnic groups overall and in each of the content strands of mathematics. White students at grade 8 also outperformed Black, Hispanic, and American Indian students in terms of overall proficiency and in each of the five content strands. At grade 4, Hispanic students performed better than Black students in Geometry and Spatial Sense, and American Indian students performed better than Black and Hispanic students in all strands. At grade 8, Hispanic students outperformed Black students in Measurement and in Geometry and Spatial Sense. At grade 12, Asian/Pacific Islander students performed better than White students in Algebra and Functions, and Hispanic students outperformed Black students in Measurement and in Data Analysis, Statistics, and Probability.

Figure 2.2

Average Proficiency in Mathematics Content Strands, Grades 4, 8, and 12

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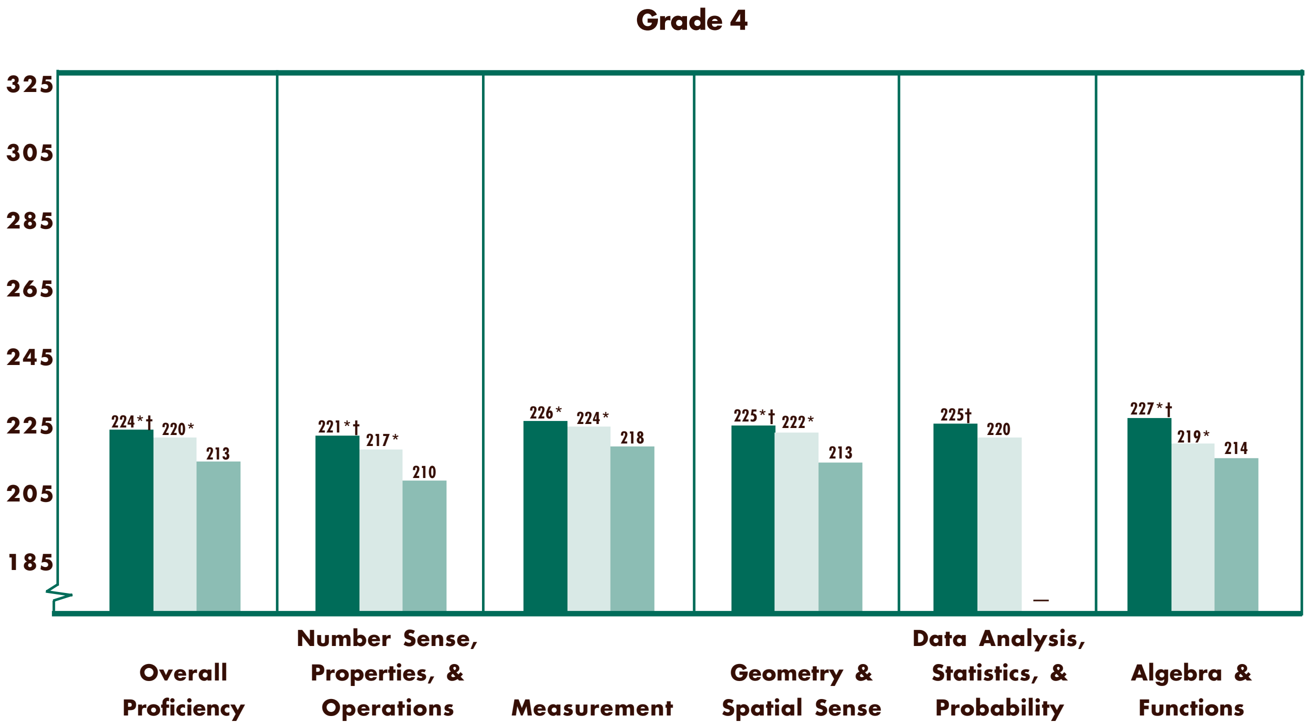

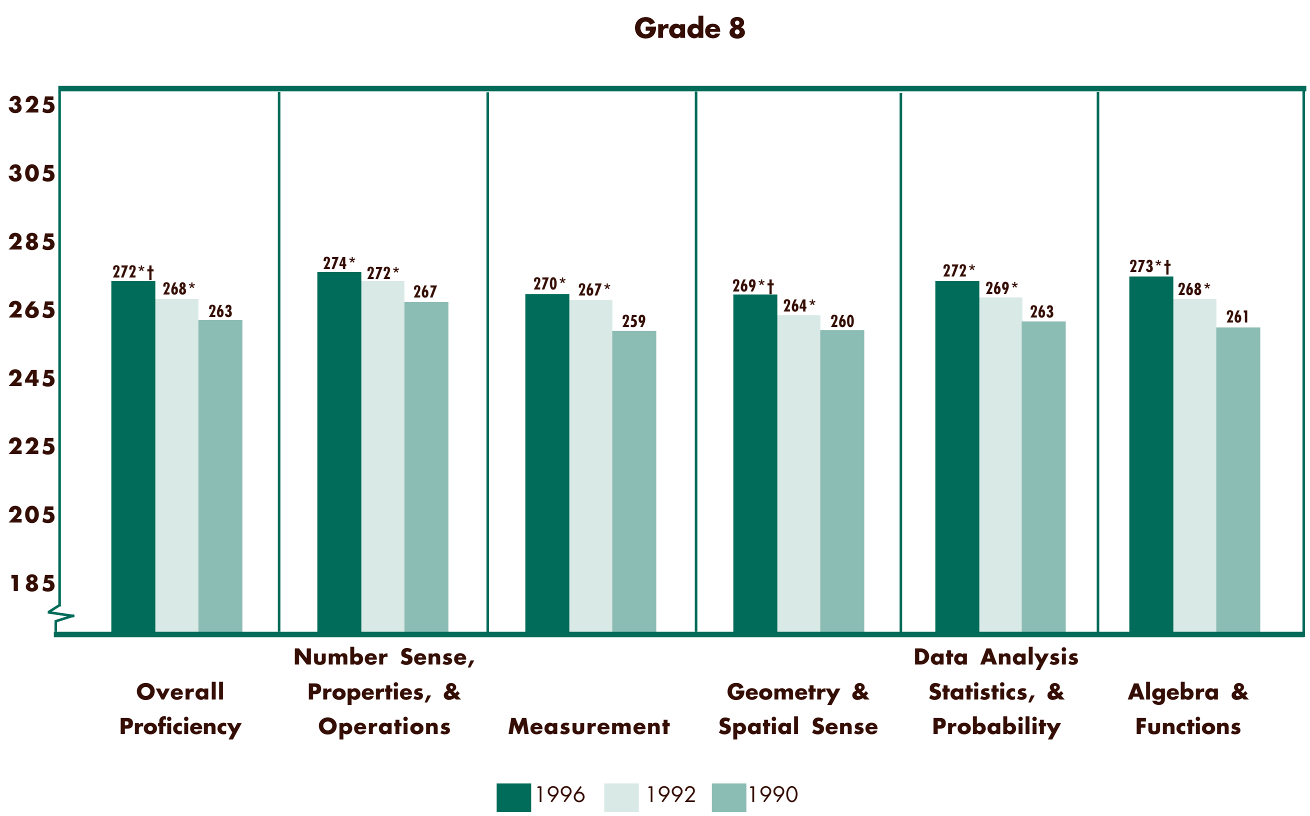

Figure 2.2 (cont)

Average Proficiency in Mathematics Content Strands, Grades 4, 8, and 12

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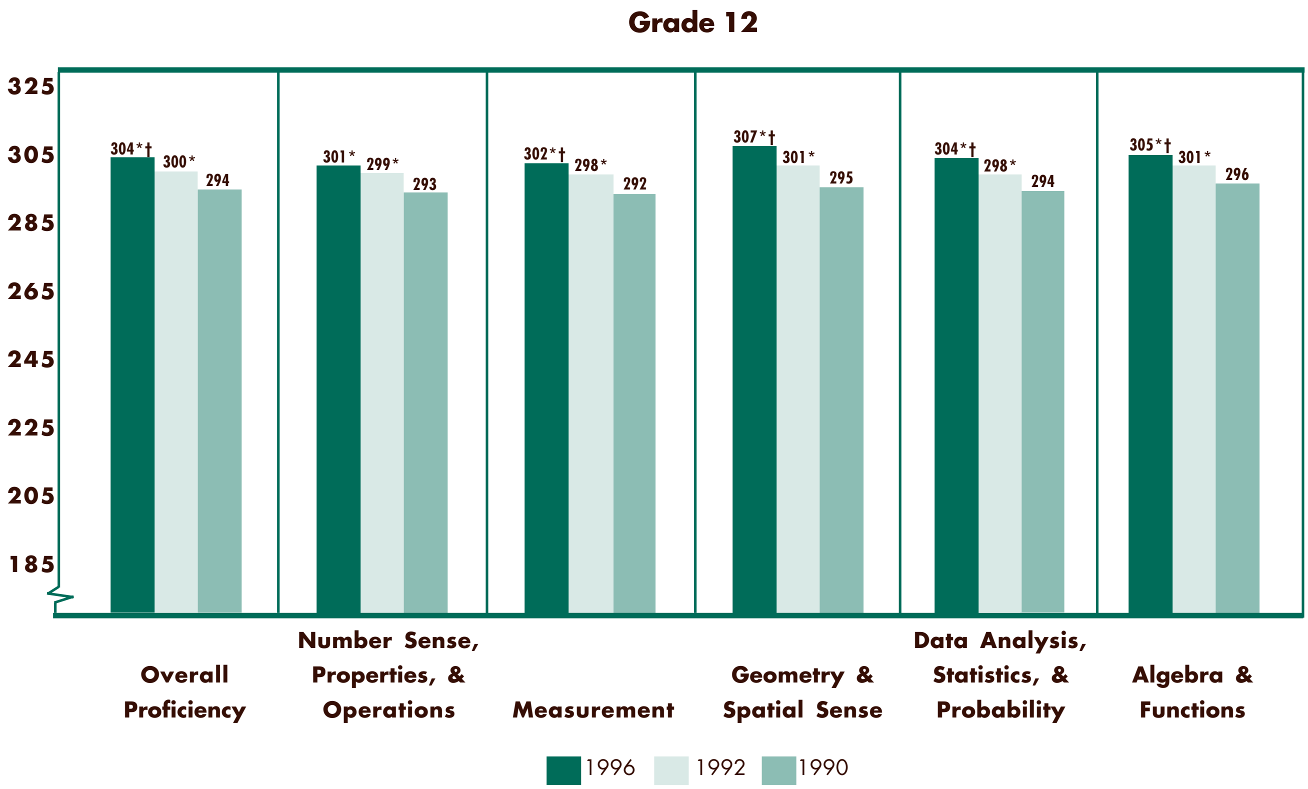

* Significant difference from 1990.

† Significant difference from 1992.

— 1990 data are not available.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Table 2.1

Average Proficiency in Mathematics Content Strands by Gender, Grades 4, 8, and 12

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	1996			1992			1990		
	All Students	Male	Female	All Students	Male	Female	All Students	Male	Female
Grade 4									
Overall Proficiency	224*†	226*†	222*†	220*	221*	218*	213	214	212
Number Sense, Properties, & Operations	221*†	223*†	220*†	217*	218*	216*	210	210	210
Measurement	226*	228*	223*	224*	226*	223*	218	221	216
Geometry & Spatial Sense	225*†	225*	225*†	222*	223*	221*	213	213	213
Data Analysis, Statistics, & Probability	225†	226†	223	220	221	220	—	—	—
Algebra & Functions	227*†	230*†	225*†	219*	218*	219*	214	214	214
Grade 8									
Overall Proficiency	272*†	272*	272*†	268*	268*	269*	263	263	262
Number Sense, Properties, & Operations	274*	274*	274*	272*	272*	273*	267	266	267
Measurement	270*	271*	268*	267*	269*	264*	259	263	255
Geometry & Spatial Sense	269*†	269*†	270*†	264*	264	264*	260	261	259
Data Analysis, Statistics, & Probability	272*	271*	274*†	269*	268	269*	263	264	263
Algebra & Functions	273*†	273*†	273*†	268*	266*	270*	261	261	262
Grade 12									
Overall Proficiency	304*†	305*†	303*†	300*	301*	298*	294	297	292
Number Sense, Properties, & Operations	301*	303*	300*	299*	300*	298*	293	296	290
Measurement	302*†	306*†	299*†	298*	302	295*	292	298	288
Geometry & Spatial Sense	307*†	309*†	305*†	301*	304*	299*	295	298	293
Data Analysis, Statistics, & Probability	304*†	304*†	304*†	298*	299	297*	294	297	292
Algebra & Functions	305*†	305*	305*†	301*	301*	300*	296	297	295

* Significant difference from 1990.
† Significant difference from 1992.
— 1990 data are not available.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure 2.3 ***Average Mathematics Proficiency, Composite Scale by Race/Ethnicity, Grades 4, 8, and 12***

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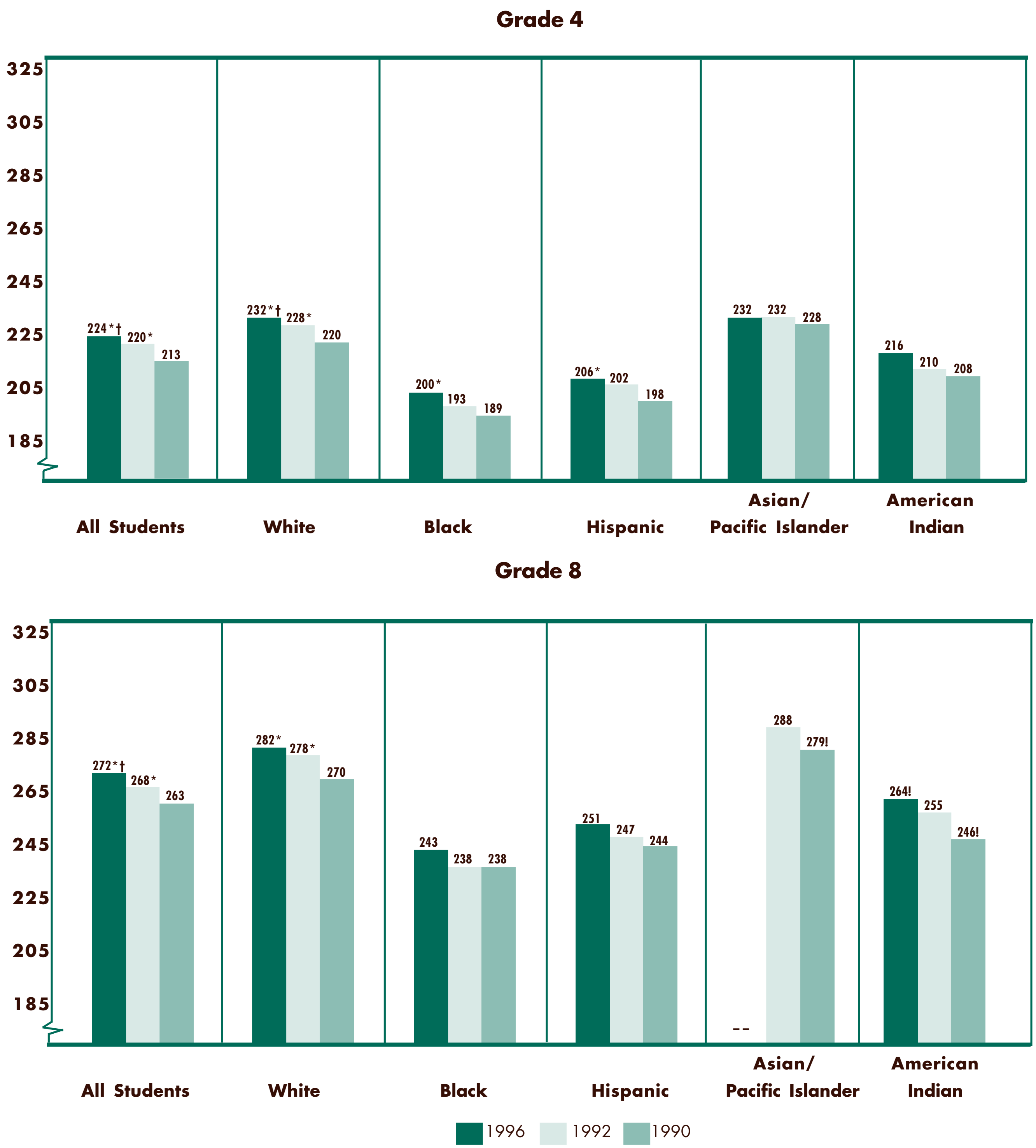
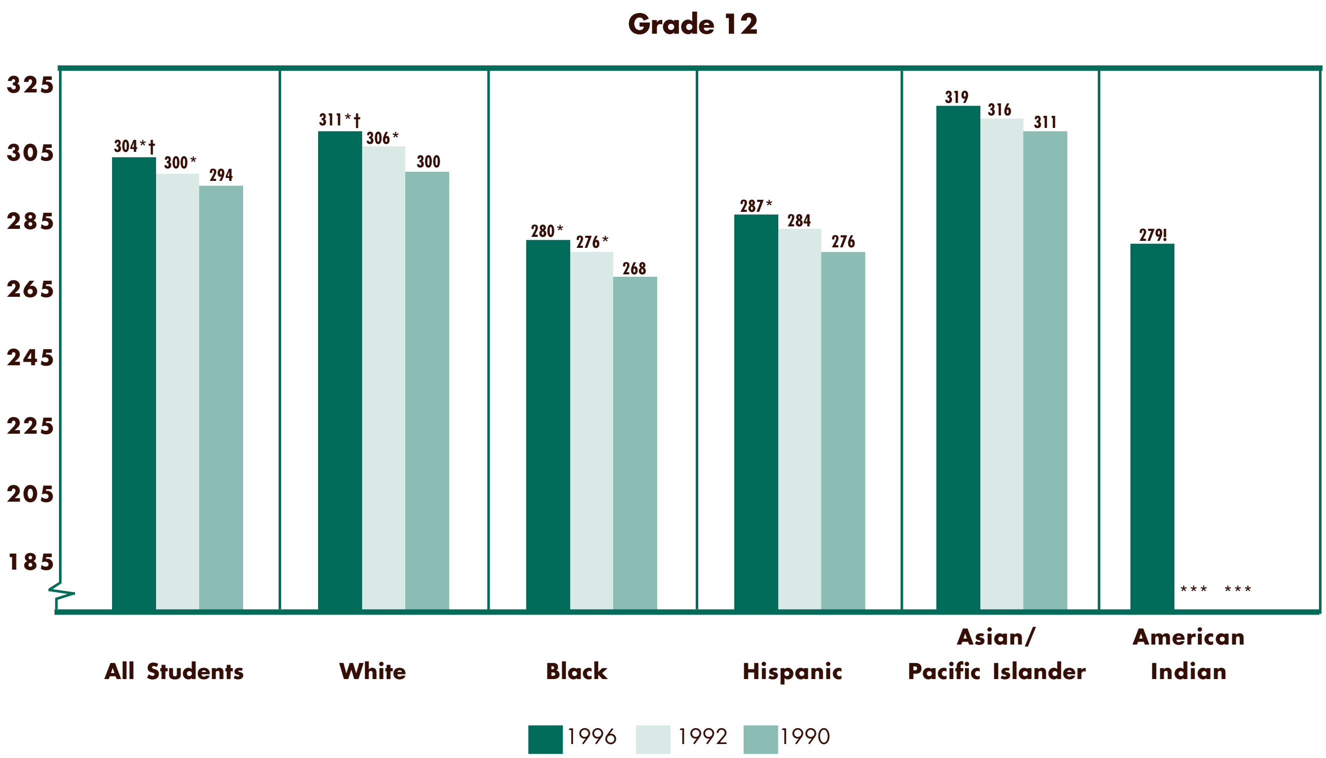


Figure 2.3 (cont)

Average Mathematics Proficiency, Composite Scale by Race/Ethnicity, Grades 4, 8, and 12



* Significant difference from 1990.

† Significant difference from 1992.

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

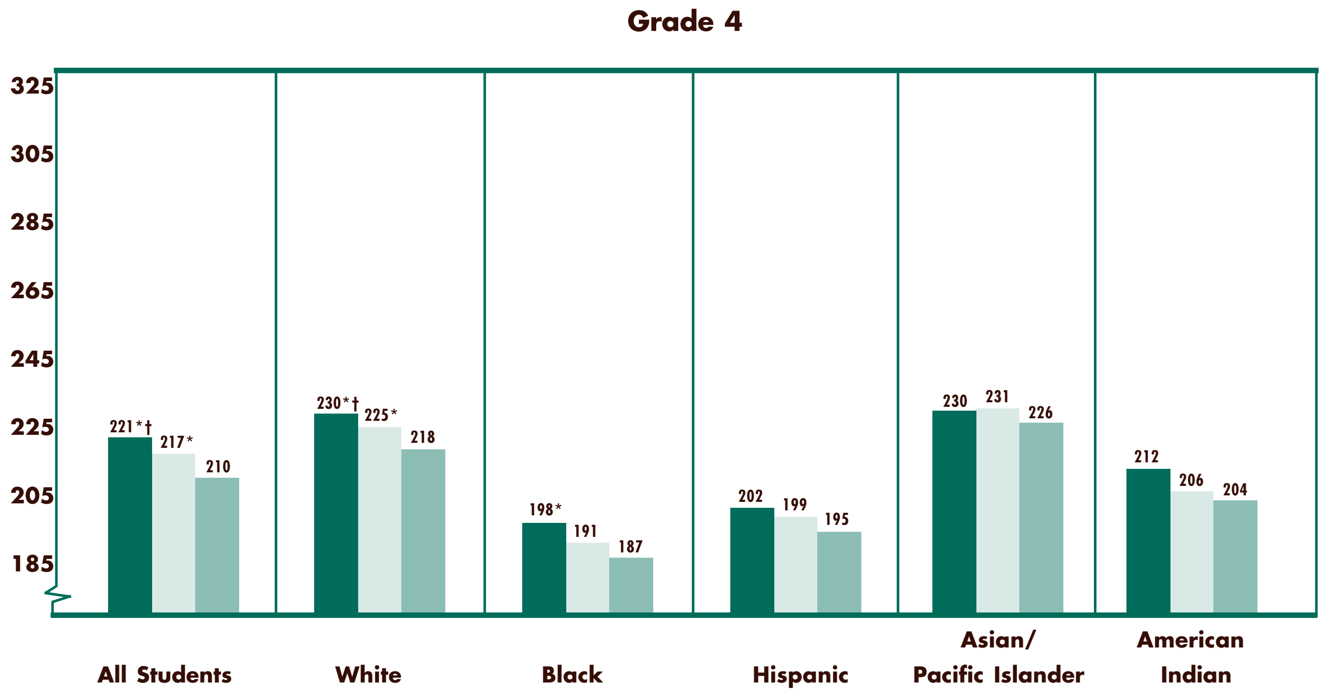
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure 2.4

Average Proficiency in Number Sense, Properties, and Operations by Race/Ethnicity, Grades 4, 8, and 12

THE NATION'S REPORT CARD naep



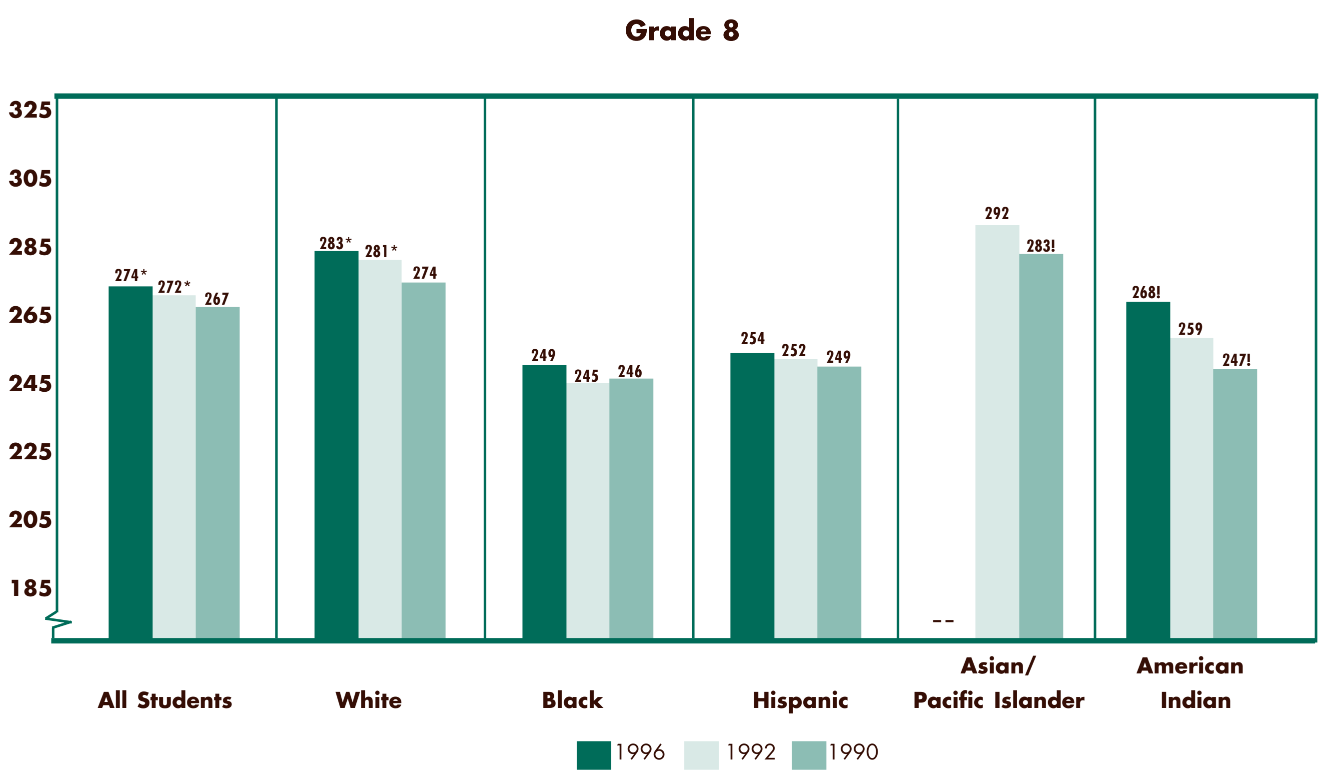
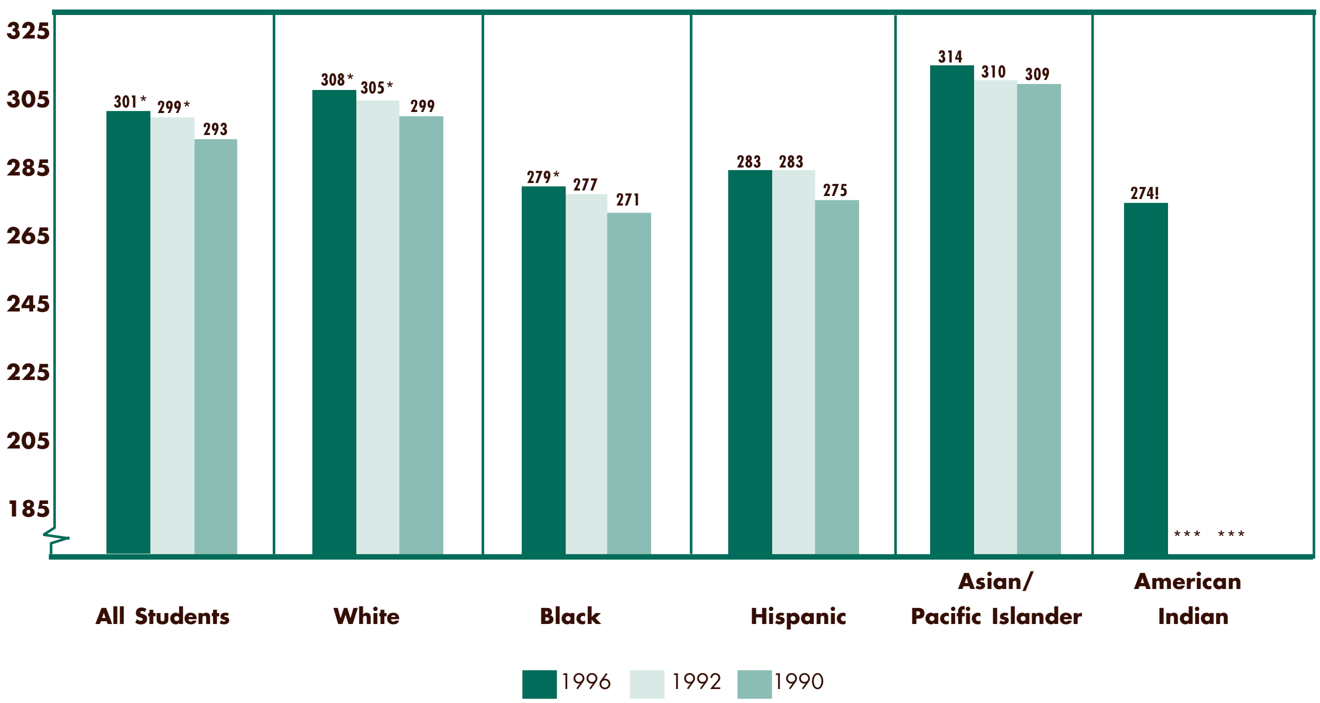


Figure 2.4 (cont)

Average Proficiency in Number Sense, Properties, and Operations by Race/Ethnicity, Grades 4, 8, and 12

THE NATION'S REPORT CARD naep

Grade 12



* Significant difference from 1990.

† Significant difference from 1992.

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure 2.5 *Average Proficiency in Measurement by Race/Ethnicity, Grades 4, 8, and 12*

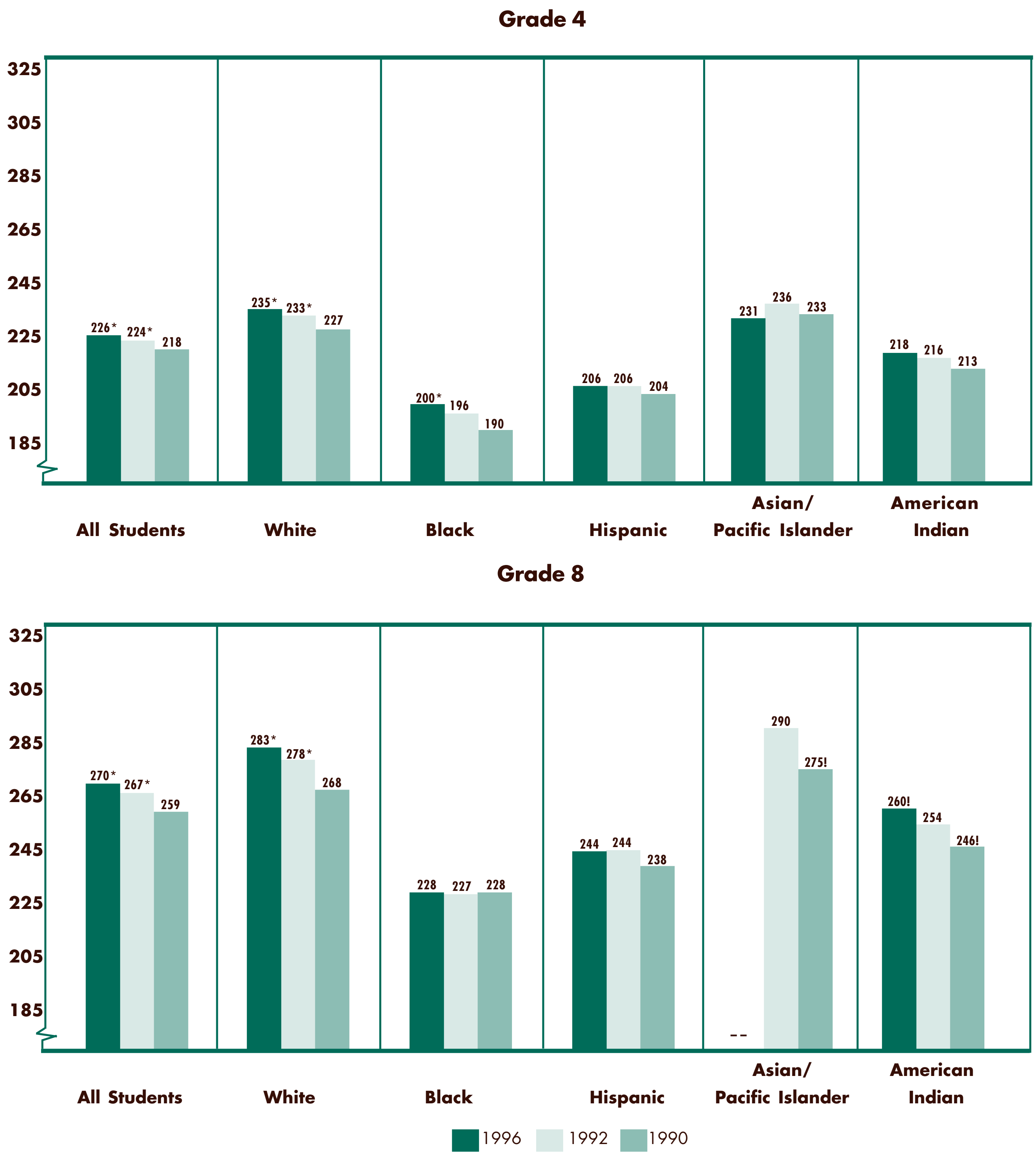
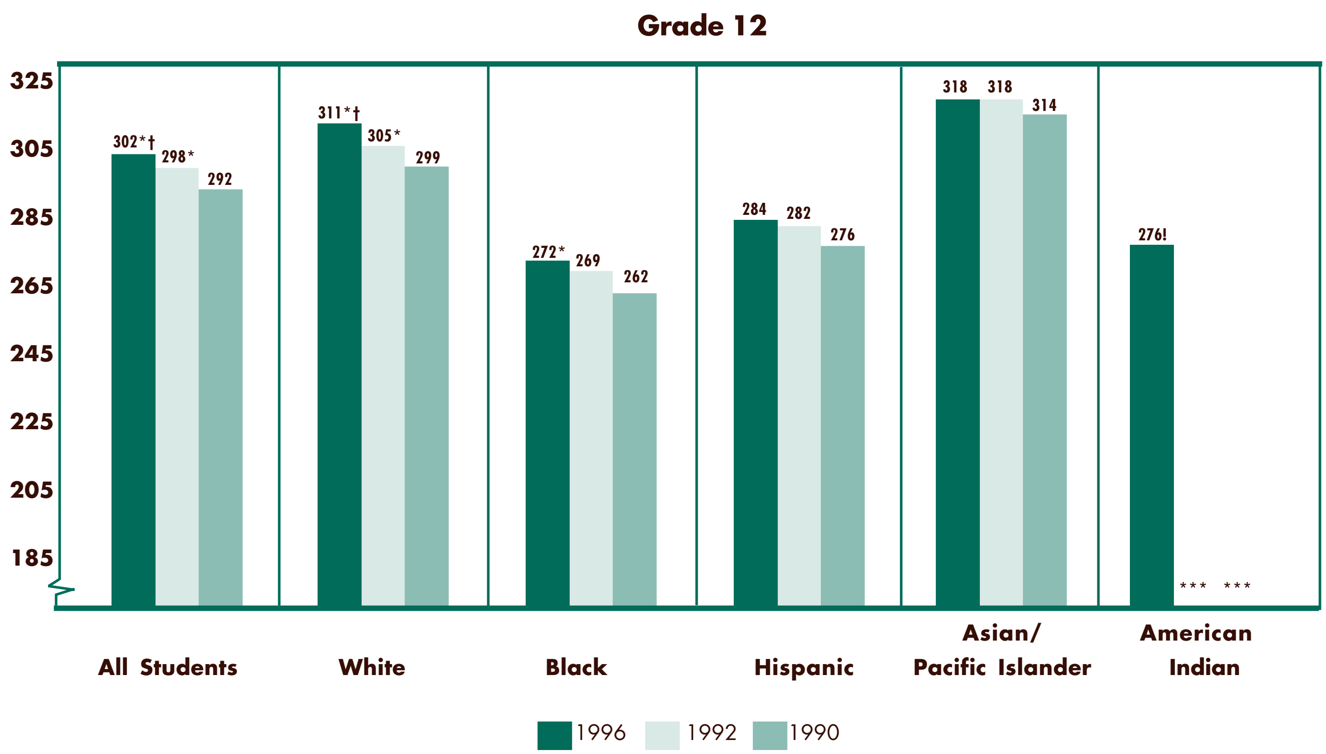


Figure 2.5 (cont)

Average Proficiency in Measurement by Race/Ethnicity, Grades 4, 8, and 12

THE NATION'S REPORT CARD naep



* Significant difference from 1990.

† Significant difference from 1992.

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure 2.6 *Average Proficiency in Geometry and Spatial Sense by Race/Ethnicity, Grades 4, 8, and 12*

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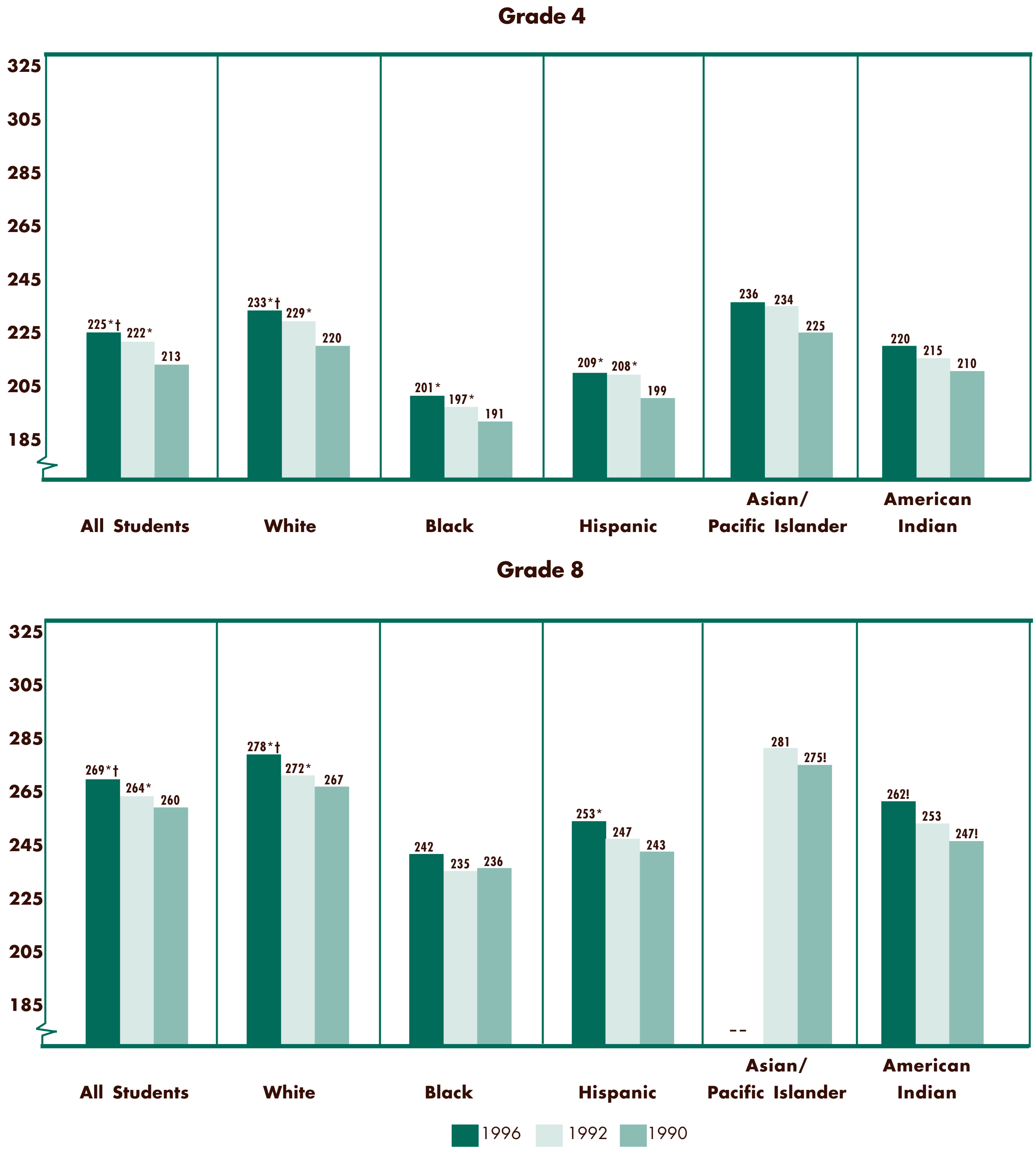
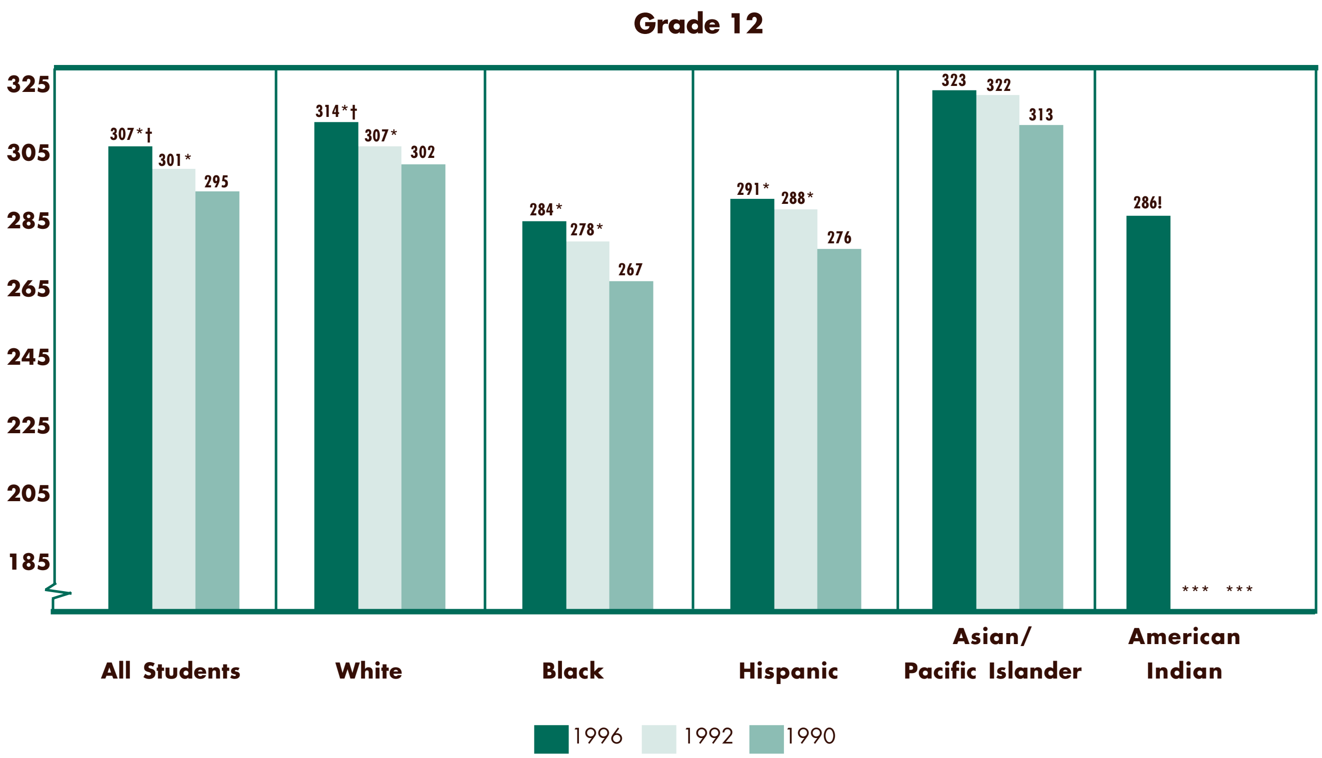


Figure 2.6 (cont) ***Average Proficiency in Geometry and Spatial Sense by Race/Ethnicity, Grades 4, 8, and 12***

THE NATION'S REPORT CARD naep



* Significant difference from 1990.

† Significant difference from 1992.

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure 2.7

Average Proficiency in Data Analysis, Statistics, and Probability by Race/Ethnicity, Grades 4, 8, and 12

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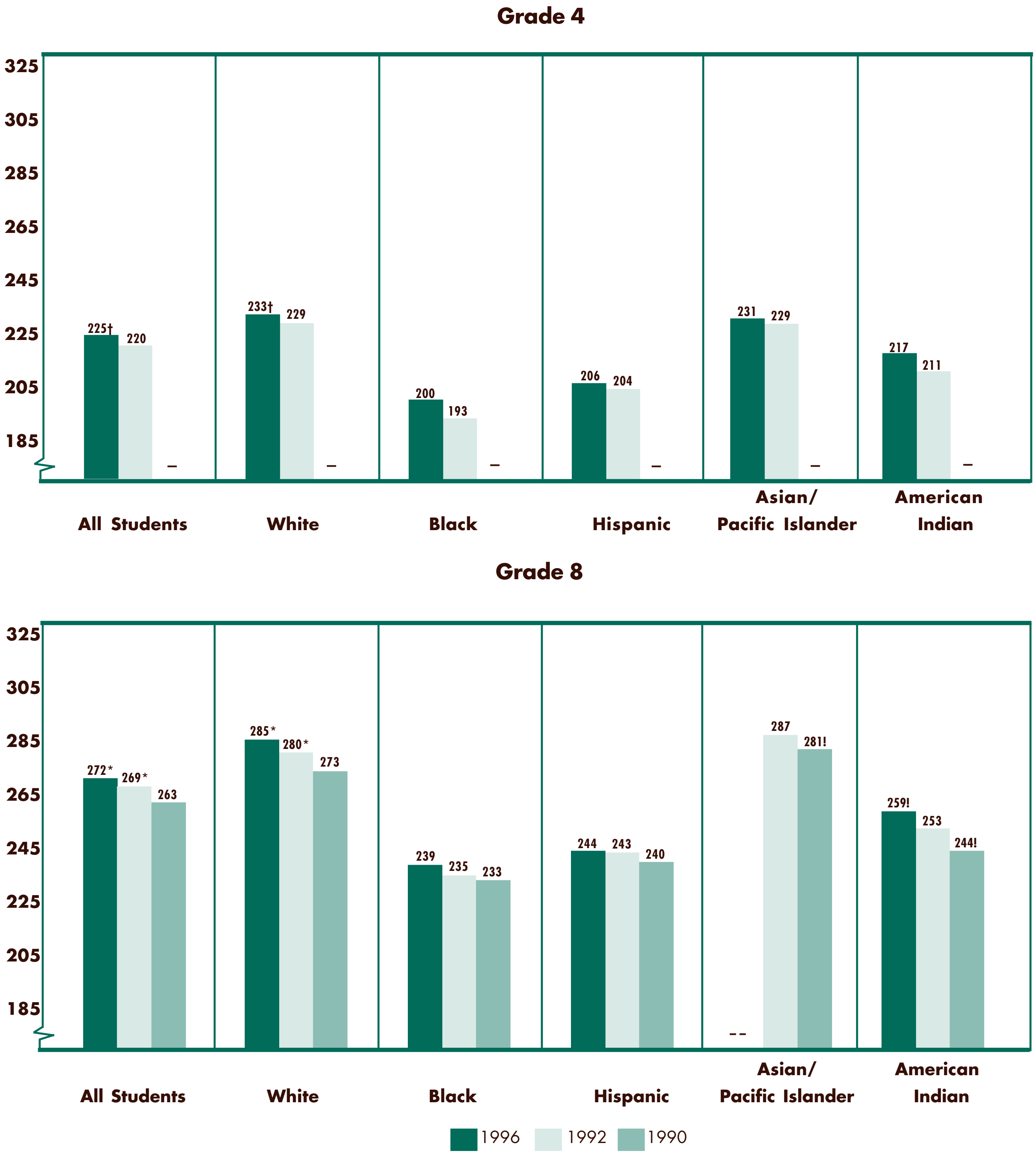

Figure 2.7 (cont)

Average Proficiency in Data Analysis, Statistics, and Probability by Race/Ethnicity, Grades 4, 8, and 12

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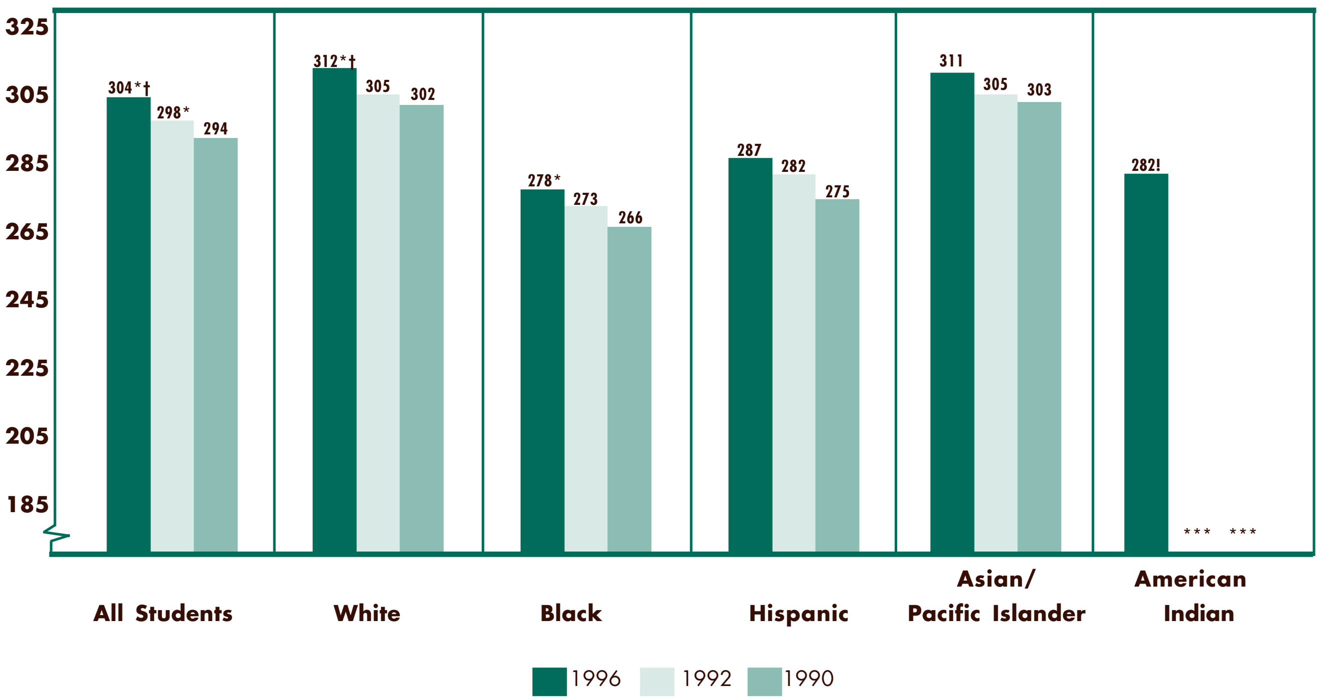
Grade 12



* Significant difference from 1990.
† Significant difference from 1992.
*** Sample size is insufficient to permit a reliable estimate.
— 1990 data are not available.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure 2.8

Average Proficiency in Algebra and Functions by Race/Ethnicity, Grades 4, 8, and 12

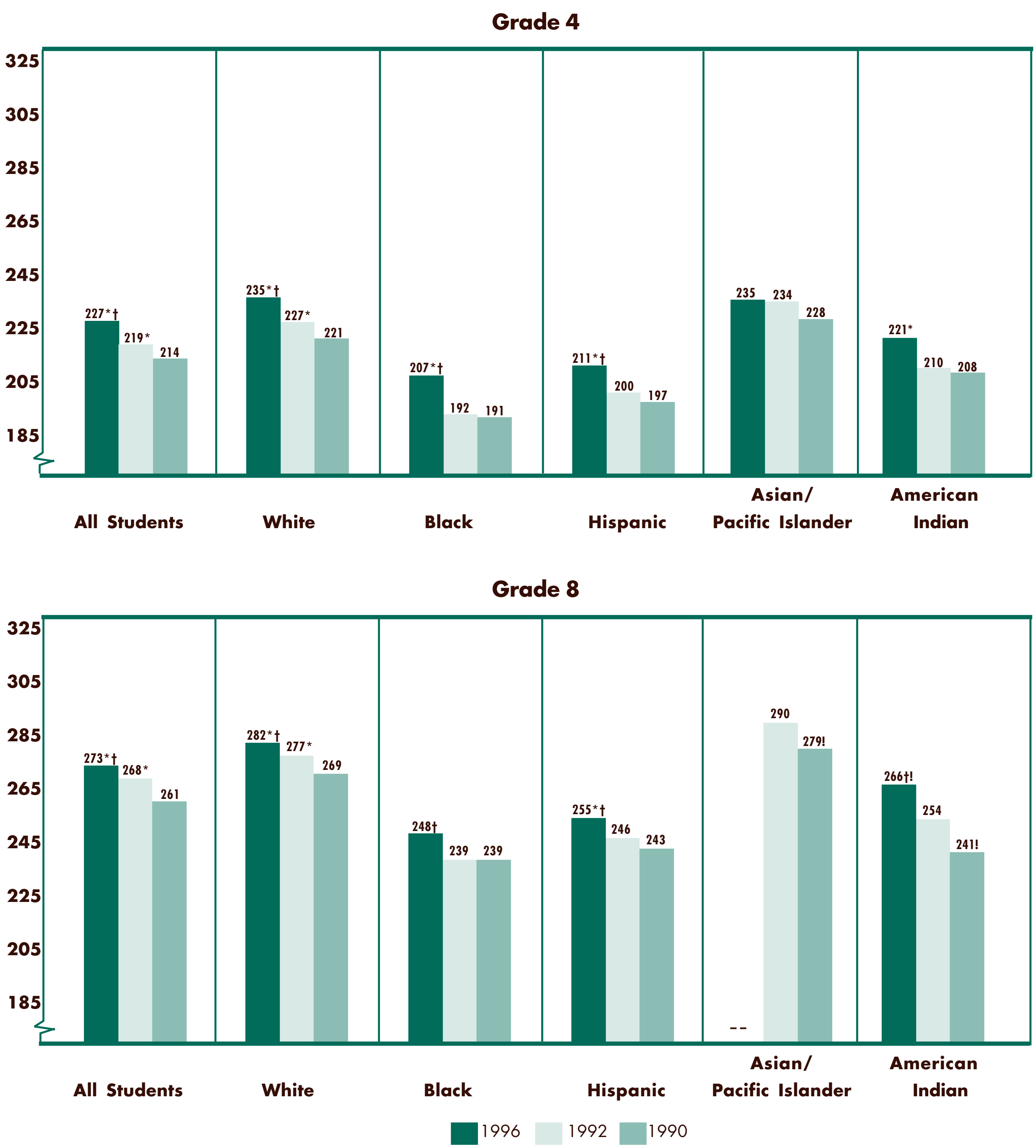
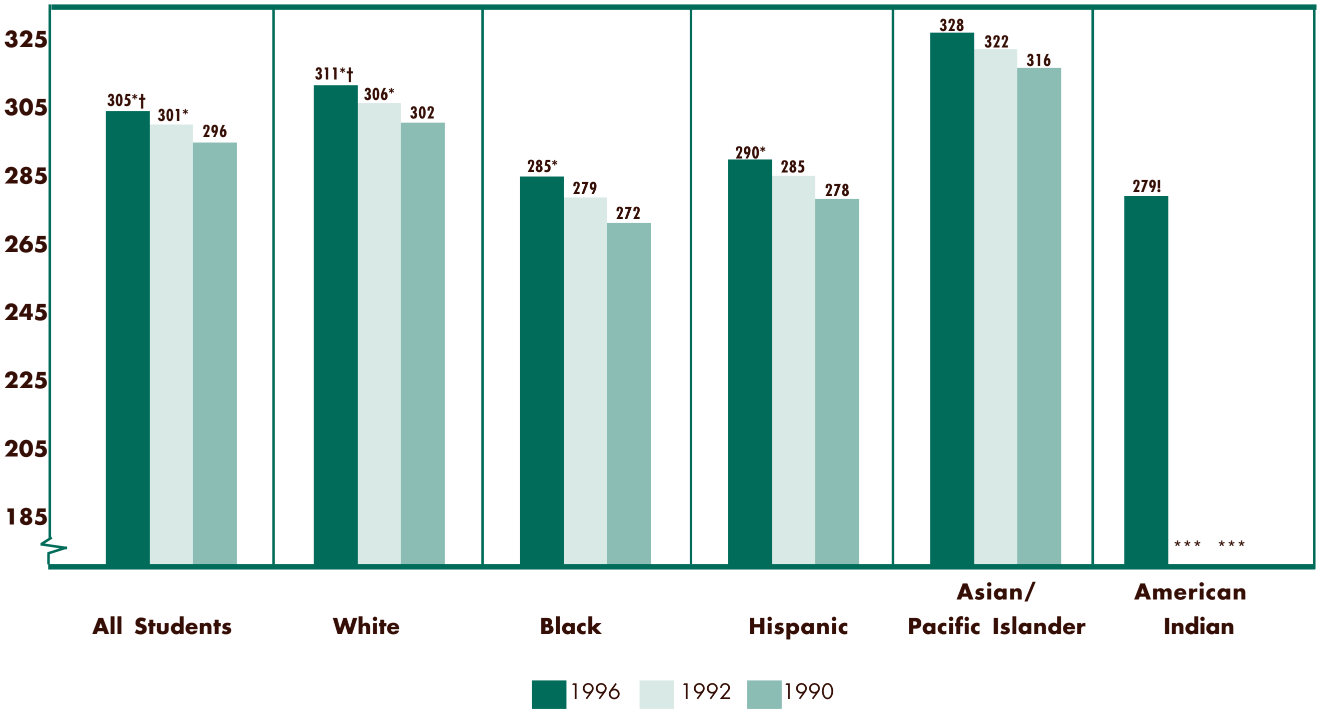


Figure 2.8 (cont)

Average Proficiency in Algebra and Functions by Race/Ethnicity, Grades 4, 8, and 12

Grade 12



* Significant difference from 1990.

† Significant difference from 1992.

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Average Proficiency in Mathematics Content Strands by Courses Taken

Performance in the different content strands is affected by the number and type of mathematics courses taken. For a discussion of the mathematics courses that eighth-grade students were enrolled in at the time of the NAEP 1996 assessment and the mathematics course-taking history of twelfth-grade students participating in the assessment, see Chapter 8. Figures 2.9–2.13 show the average 1996 performance on content strands for eighth- and twelfth-grade students with different course-taking patterns. In general, taking more mathematics courses and more advanced mathematics courses was associated with improved mathematics performance in all content strands. Figure 2.9 shows average scale scores in each content strand for eighth-grade students enrolled in algebra, pre-algebra, or eighth-grade mathematics. Eighth-grade students enrolled in algebra performed better in all content strands than eighth-grade students enrolled in pre-algebra or eighth-grade mathematics. Similarly, eighth-grade students enrolled in pre-algebra performed better than students enrolled in eighth-grade mathematics in all content strands except Geometry and Spatial Sense. In the latter content strand, performance was not significantly different for the two groups.

Twelfth-grade results show a similar story. Figure 2.10 presents average scale scores for each content strand for twelfth-grade students according to the highest course they had taken in the algebra-through-calculus sequence.[3] The algebra-through-calculus sequence consists of elementary and intermediate algebra, followed by pre-calculus and calculus. Students at any given point in this sequence performed better than students whose mathematics exposure had stopped at the next lowest course in the sequence. The only exception was that students whose highest course had been pre-algebra did not score significantly higher than students who had taken neither pre-algebra nor algebra. There was no significant difference in the performance of these two groups of students.

Figures 2.11 and 2.12 report average scale scores for students who had taken at least one course in geometry or in probability or statistics, as well as for students who had not taken these courses. The results in Figure 2.11 show that students who had taken geometry performed better in all content strands than those who had not taken geometry. This overall higher performance might be explained by the fact that most of the students who took geometry also took at least 2 years of algebra, whereas most of the students who did not take geometry took 1 year or less of algebra. The performance results were different, however, when comparing students who had taken a course in probability or statistics with those who had not (Figure 2.12). Here, there were no significant differences between the two groups. It may be that students who take probability or statistics are students who do not take more advanced courses, or that the assessment questions did not provide sufficient breadth to allow students taking statistics courses to display their added knowledge.

[3] The twelfth-grade course sequence was defined as the algebra-through-calculus sequence, not including geometry, because variability in mathematics course sequencing from school to school makes the position of geometry in the curriculum ambiguous.

The final figure in this section (2.13) shows that taking more mathematics courses in high school is related to higher mathematics performance. (The only exception was in the Measurement content strand, where the apparent difference between students who took 3 to 4 semesters of mathematics and those who took only 1 to 2 semesters was not statistically significant.)

Figure 2.9 ***Average Proficiency in Mathematics Content Areas by Course Taking, Grade 8***

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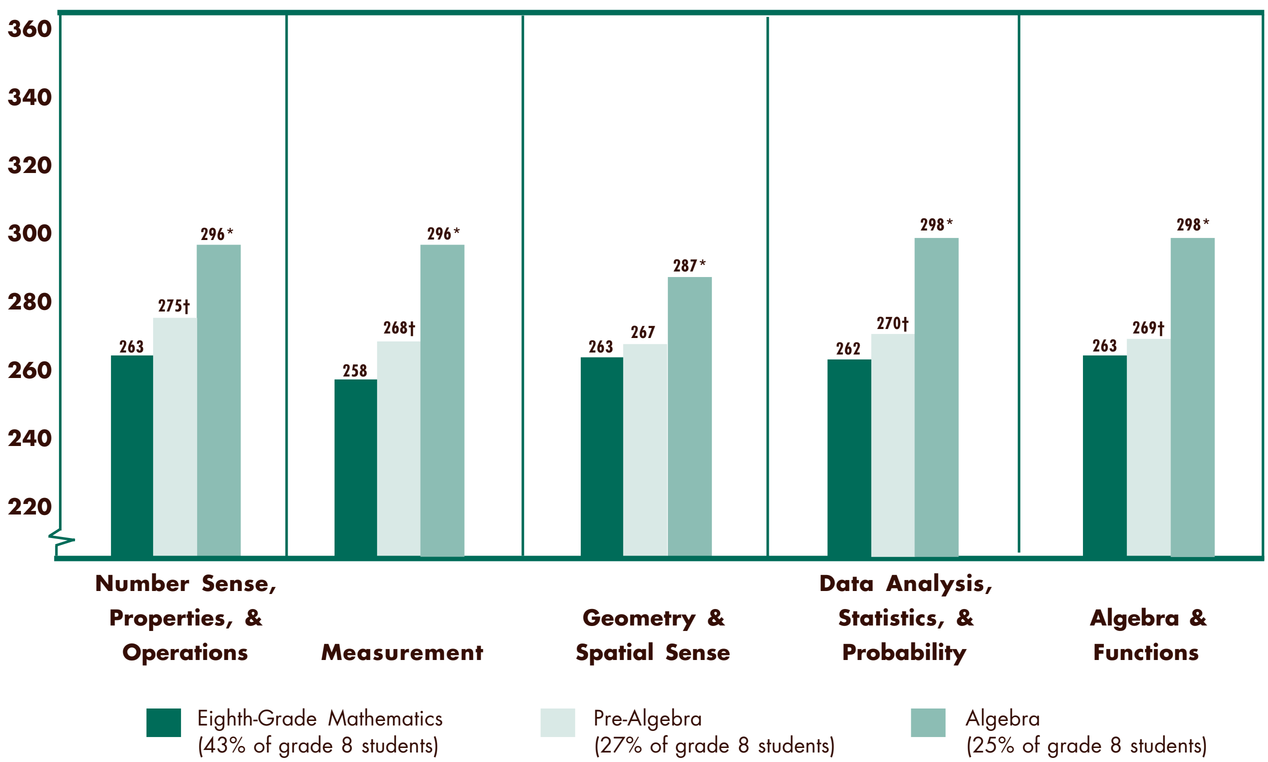

* Indicates a significant difference between algebra and pre-algebra group results and between algebra and eighth-grade mathematics group results.

† Indicates a significant difference between pre-algebra and eighth-grade mathematics group results.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Figure 2.10

Average Proficiency in Mathematics Content Areas by Algebra and Calculus Courses Taken, Grade 12*

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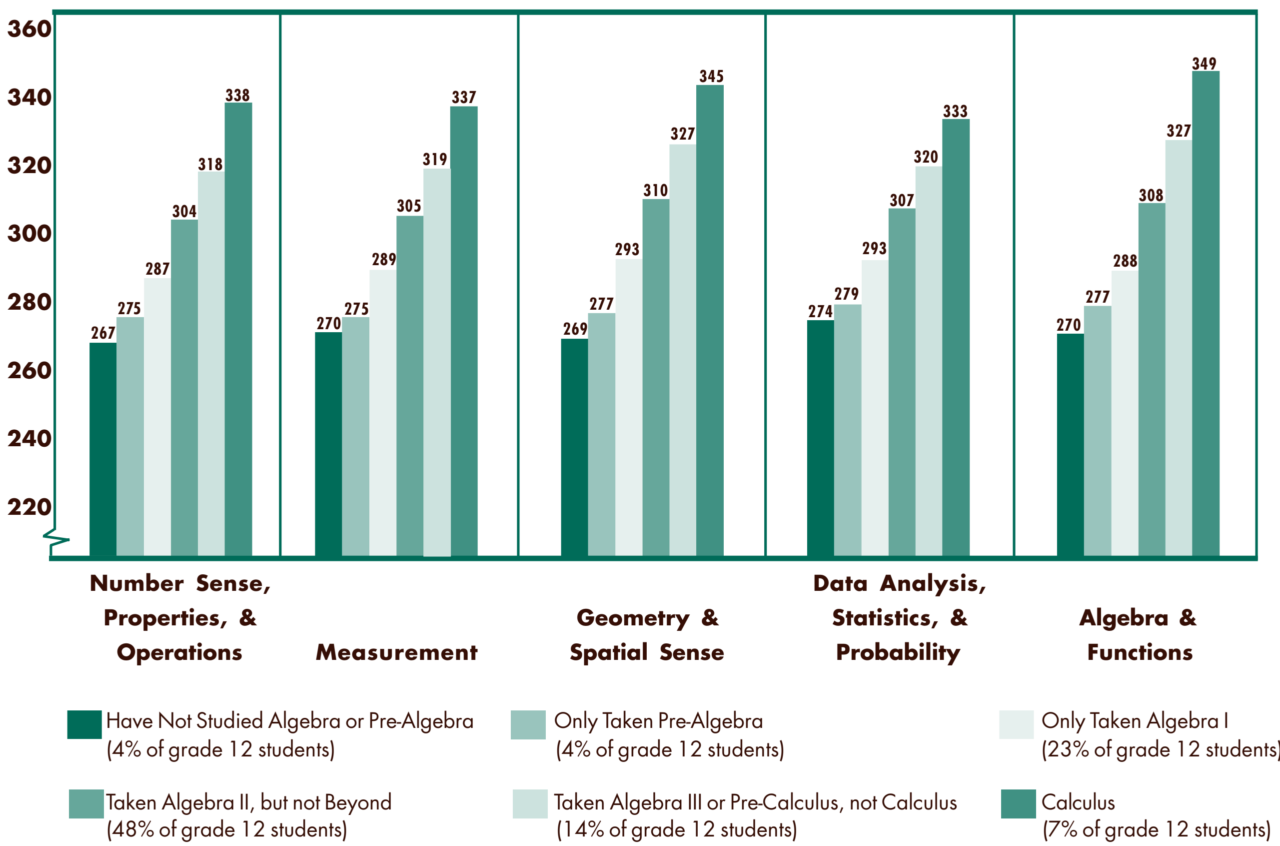

* Students at any given point in this sequence performed significantly better than students whose mathematics exposure had stopped at the next lowest course in the sequence. The only exception was that students whose highest course had been pre-algebra did not score significantly higher than students who had taken neither pre-algebra nor algebra.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Figure 2.11 ***Average Proficiency in Mathematics Content Areas by Geometry Course Taken, Grade 12***

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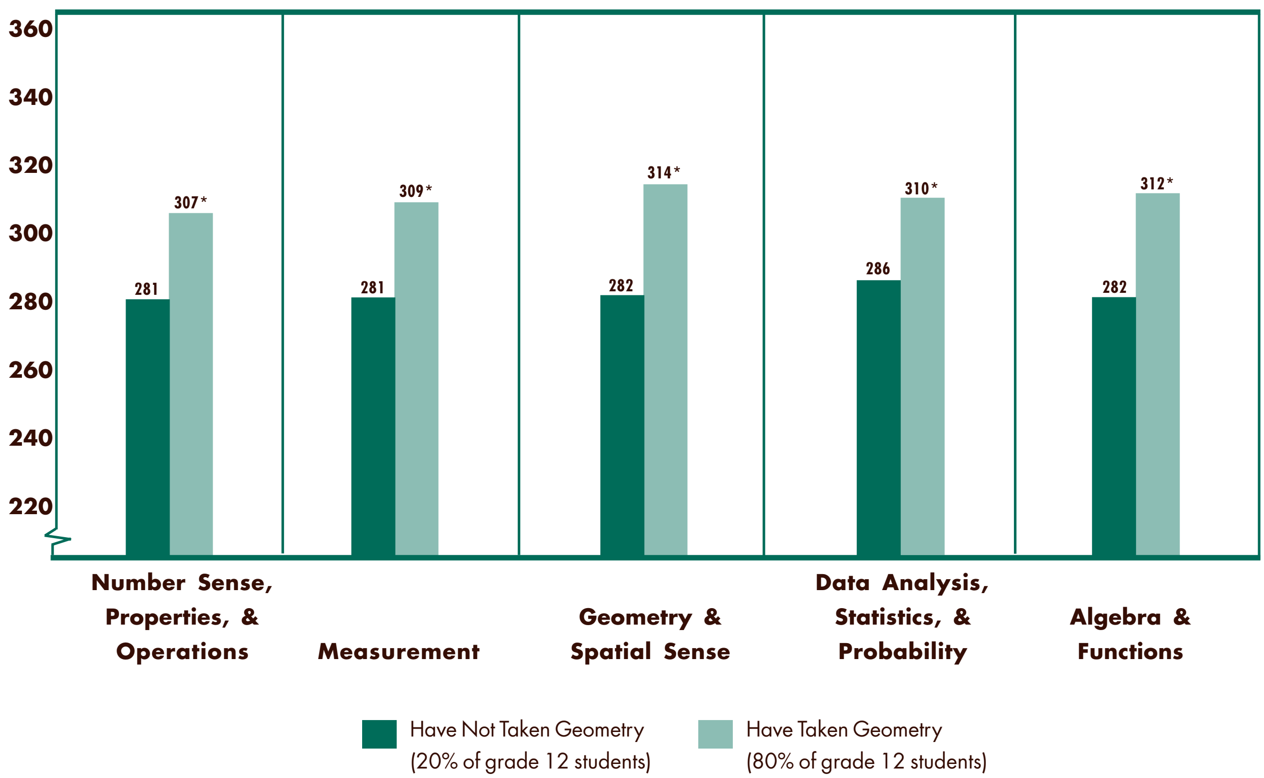

* Indicates a significant difference in results between those who had taken geometry and those who had not taken geometry.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Figure 2.12

Average Proficiency in Mathematics Content Areas by Probability or Statistics Course Taken, Grade 12

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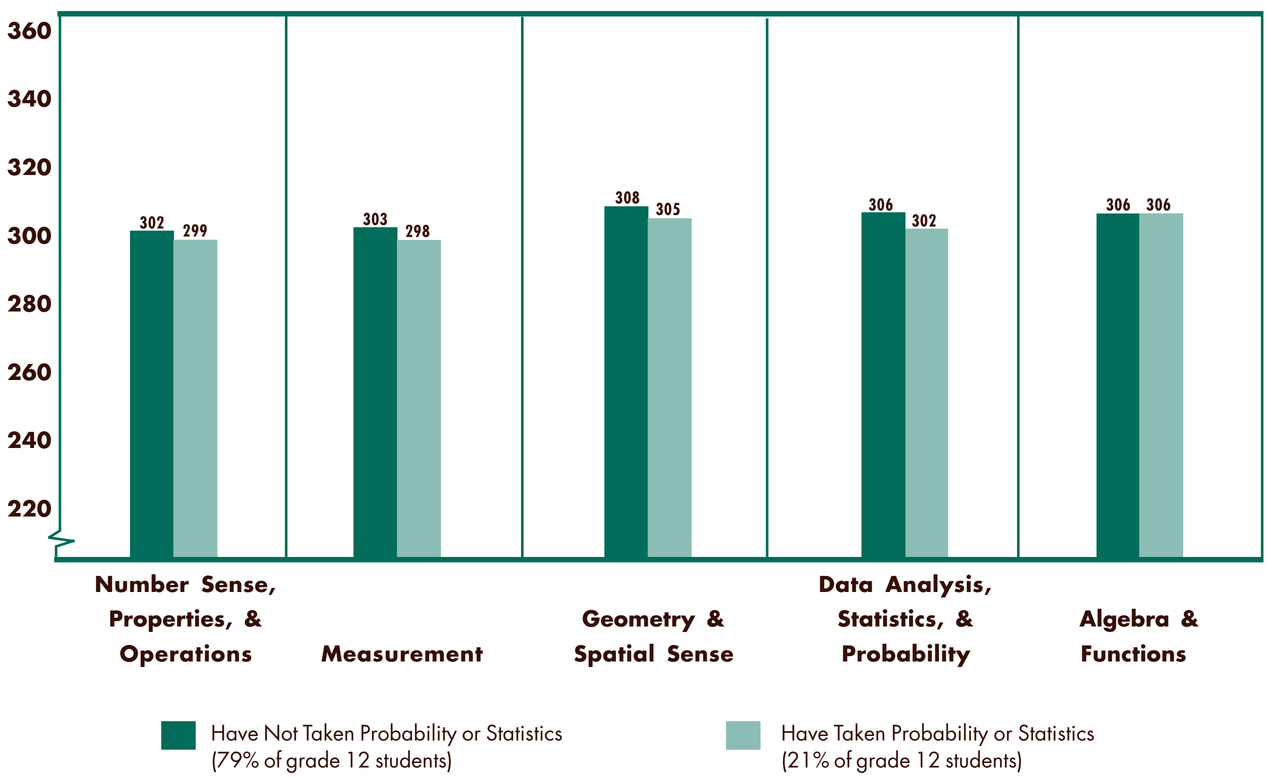
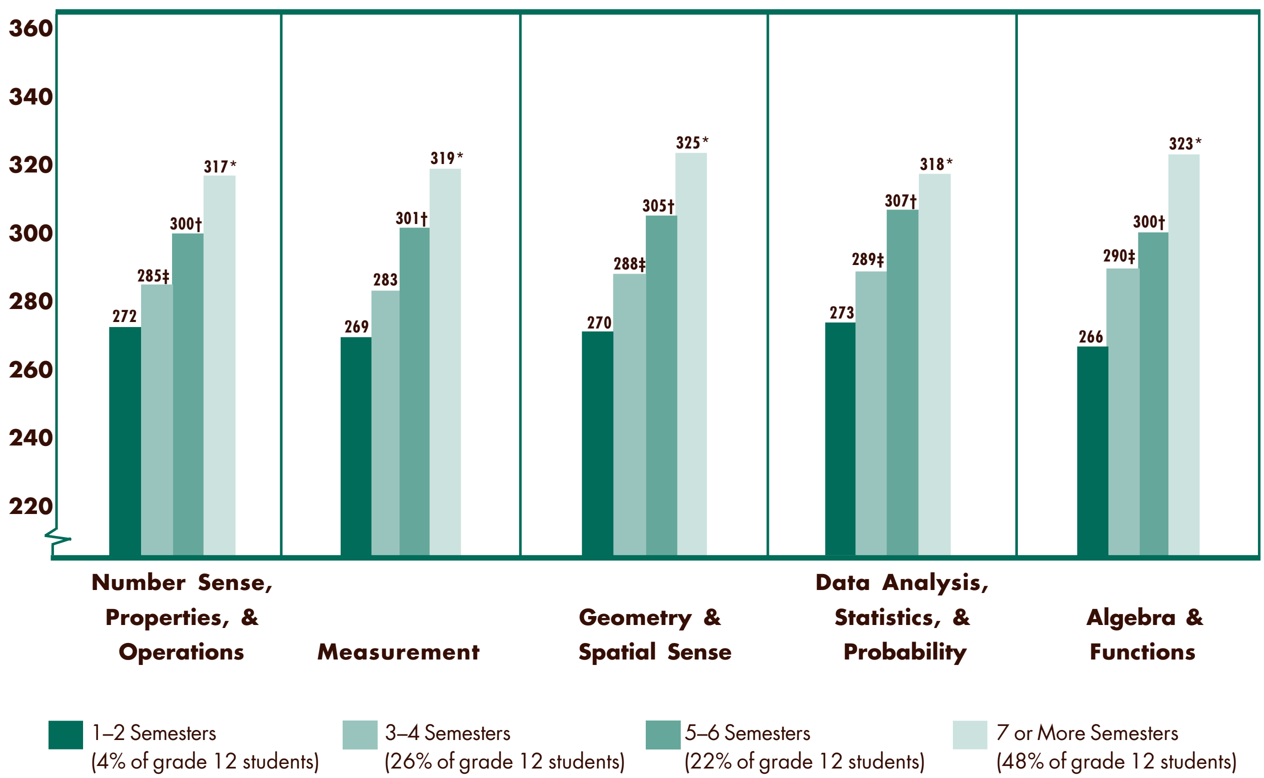


SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Figure 2.13

Average Proficiency in Mathematics Content Areas by Number of Semesters of Mathematics Courses Taken in Grades 9 through 12, Grade 12



NOTE: Sample size for 0 semesters is insufficient to permit a reliable estimate.

* Indicates a significant difference between results for students with 7 or more semesters and students with 5–6 semesters of mathematics.

† Indicates a significant difference between results for students with 5–6 semesters and students with 3–4 semesters of mathematics.

‡ Indicates a significant difference between results for students with 3–4 semesters and students with 1–2 semesters of mathematics.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Introduction to Content Strand Chapters

So far, this report has provided an overall look at performance trends in the five mathematics content strands. Chapters 3–7 provide a detailed look at student performance *within* each of the five content strands and offer many examples of assessment questions and actual student work. The goal of Chapters 3–7 is to provide the reader with a general understanding of a) the range of skills assessed within each content strand, b) how 1996 performance varied within each content strand by grade, and c) how performance varied across grades. When reading these chapters, it is important to bear in mind the variety of factors that influence student performance and, therefore, the relative difficulty of particular questions. One source of performance variation is content, represented in the NAEP 1996 mathematics assessment by the content strands. Students' opportunities to learn content vary, as does their ability to retain what they have learned and to apply their content knowledge to assessment questions. However, other sources of performance variation are related to the questions themselves, irrespective of the content. Some of these sources — the type of question (e.g., multiple-choice, constructed-response), the extent to which the question draws upon the different mathematical abilities (e.g., conceptual understanding, procedural knowledge, problem solving) and mathematical power (e.g., communication, reasoning), and the use of manipulatives — were described in Chapter 1. Other characteristics of questions, such as how the question is presented (e.g., as a real-life problem, as a numerical equation, with a pictorial or graphical representation) and the number of steps required to reach a solution, also influence performance. Thus, two questions may be assessing the same content but may elicit different results by virtue of how the problem was presented to the students, what students were asked to do with the content, and how students were asked to respond. Understanding how and why student performance varies, therefore, entails more than just knowing what content a question was measuring; it also entails knowing *how* the question was measuring what it measured.

In order to provide a better understanding of the multiple ways in which different item characteristics can be combined, Table 2.2 lists a few questions that appear as examples in the chapters that follow. The device of "map number," used in the table, is described in more detail below, but questions with higher map numbers are generally more difficult than questions with lower numbers.

In each of the chapters that follow, several questions have been selected from a set of released questions to illustrate what is assessed in each content strand. To provide the reader with a ready visual reference for the relative difficulty of the questions, they have been mapped onto the NAEP composite mathematics scale, which is the measure of overall mathematics achievement. For each question, the item map provides a marker of the performance level at which students are relatively likely to answer the question correctly.[4] The questions for all three grade levels map onto the composite scale, whose possible values range between 0 and 500. Most fourth-grade questions map to the lower end, most eighth-grade questions map to the middle, and most twelfth-grade questions map to the higher end of the scale. Some questions

[4] The procedures used to develop the item maps are detailed in Allen, N. L., Carlson, J. E., & Zelenak, C. A. (1999). *The NAEP 1996 technical report*. Washington, DC: National Center for Education Statistics.

were administered to students at more than one grade level and may map at slightly different values for each grade. That is, mathematics skills are generally cumulative in nature, justifying the use of a single scale to portray the growth in mathematics achievement across years. The relationship between performance on a specific question and overall mathematics performance, however, may not be entirely consistent across grades.

Table 2.2 **Characteristics of Sample Questions from the NAEP 1996 Mathematics Assessment**

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Question	Map	Grade	Content Strand	Question Type[a]	Ability[b]	Power[c]	Other
Relate a Fraction to 1	248	4	Num.	SCR	CU		
Describe Measurement Task	332	4	Meas.	SCR	PS	RE/CM/CN	real-life problem
Compare Two Geometric Shapes	324	4	Geo.	ECR	PS		question contains a picture
Translate Words to Symbols	281	8	Alg.	MC	CU		real-life problem
Subtract Integers	335	8	Alg.	SCR	PK	RE/CN	real-life problem, multistep
Compare Mean and Median	463	12	Data	ECR	PS	RE/CM/CN	question contains a table
Draw a Parallelogram with Perpendicular Diagonals	356	12	Geo.	SCR	PS		solution requires drawing

[a] MC = Multiple-Choice, SCR = Short Constructed-Response, ECR = Extended Constructed-Response.

[b] CU = Conceptual Understanding, PK = Procedural Knowledge, PS = Problem Solving.

[c] RE = Reasoning, CN = Connecting, CM = Communicating.

To see how to interpret the item map, consider the first question in the table above, "Relate a Fraction to 1." This is a short constructed-response question for fourth graders that is scored right/wrong. It maps at a scale score of 248 (refer to item map, Figure 2.14). Mapping the question at a score of 248 on the NAEP composite mathematics scale implies that students whose overall mathematics proficiency scores are 248 or higher have at least a 65 percent chance of correctly answering this question.[5] Students whose overall mathematics proficiency scores are below 248 have less than a 65 percent chance of correctly answering the question. This does not mean that students at or above the 248 level always answer this question correctly, or that students below the 248 level always answer the question incorrectly. Rather, students have a higher or lower probability of successfully answering the question depending on their overall ability as measured on the NAEP mathematics scale.

[5] For constructed-response questions, a criterion of 65 percent was used. For multiple-choice questions with four or five alternatives, the criteria were 74 and 72 percent, respectively. The use of a higher criterion for multiple-choice questions reflects students' ability to guess the correct answer from among the alternatives.

Note that the item mapping refers to student performance on the *composite* mathematics scale (i.e., the scale for general mathematics performance) and not to performance on the separate scales for each content strand. Thus, in the example above, the map number for the question, which is from the Number Sense, Properties, and Operations content strand, locates the question in relation to the performance of fourth-grade students on the entire mathematics assessment, as opposed to their performance on the Number Sense, Properties, and Operations content strand alone.

Chapters 3–7 discuss student performance within each of the five content strands in more detail. Several assessment questions from the content strands of the NAEP 1996 assessment are shown in these chapters. The sample questions were selected from those released at each of the three grade levels to illustrate varying difficulty levels, a variety of question formats, and different mathematical abilities. Examples also were chosen to illustrate how questions tested students' conceptual understanding and procedural knowledge, as well as their abilities to reason, communicate, and make connections.

In each chapter, the content in a particular content strand is further organized into areas for ease of presentation. The organizing areas are not mutually exclusive, and many questions required students to use knowledge and skills from more than one area. At least one sample question is presented for each area, and information about students' performance on each sample question is given for all students, as well as for students classified by gender and race/ethnicity. For questions on the eighth-grade assessment, student performance also is examined with respect to the mathematics course students currently are taking. For questions on the twelfth-grade assessment, performance is examined with respect to the highest mathematics course students have taken in the algebra-through-calculus sequence.

The impact of taking geometry on student performance at the twelfth-grade level is not discussed, although the data are presented in the tables. Because more able students are likely to progress further in the mathematics course sequence, it is difficult to separate the impact of a particular curriculum from the impact of the student's overall strengths in mathematics. In addition, the pool of students on which the specific influence of geometry could be isolated is rather small: 90 percent of students report having 1 year or more of first-year algebra, 80 percent report having 1 year or more of geometry, and 70 percent report having 1 year or more of second-year algebra. Therefore, on the assumption that second-year algebra typically follows geometry, only about 10 percent of students could be classified as having had first-year algebra and geometry, but no further mathematics. For these reasons, discussion of the impact of mathematics courses on student performance at twelfth grade is limited to the algebra-through-calculus sequence.

Figure 2.14

Map of Selected Questions on the NAEP Composite Mathematics Scale (Item Map)

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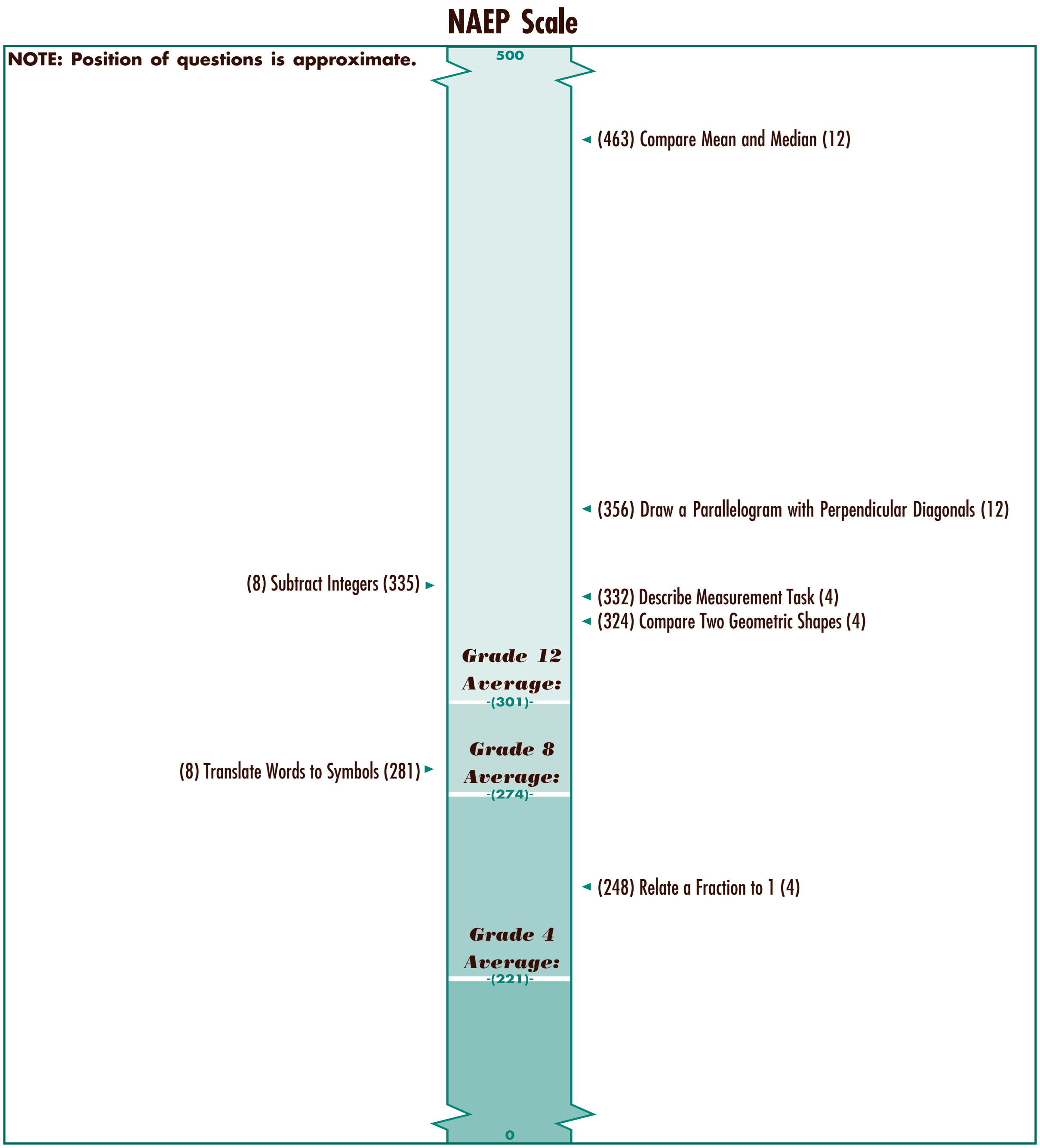


NOTE: Each mathematics question was mapped onto the NAEP 0 to 500 mathematics scale. The position of the question on the scale represents the scale score obtained by students who had a 65 percent probability of successfully answering the question. (The probability was 74 percent for a 4-option multiple-choice question and 72 percent for a 5-option multiple-choice question.) Only selected questions are presented. The number 4, 8, or 12 in parentheses is the grade level at which the question was asked.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Chapter 3

Number Sense, Properties, and Operations

Content Strand Description

The questions included in the content strand of Number Sense, Properties, and Operations primarily covered basic arithmetic skills and concepts. These skills and concepts represent a significant part of the mathematics curriculum, particularly at the lower grade levels, at most American schools. Reflecting this emphasis, a large portion of the questions on the NAEP 1996 mathematics assessment fell under this content strand, although the portion was smaller than in 1990 and 1992. As shown in Table 1.1 in Chapter 1, 40 percent of the mathematics questions given to fourth-grade students, 25 percent of those given to eighth-grade students, and 20 percent of those given to twelfth-grade students fell into this content strand.

The Number Sense, Properties, and Operations content strand focused on students' understanding of numbers (whole numbers, fractions, decimals, integers, real numbers, and complex numbers), operations, and estimation, and on application of their understanding to real-world situations. Questions in this content strand required students to demonstrate an understanding of number properties and operations, to generalize from numerical patterns, and to verify results. The questions also assessed student understanding of numerical relationships as expressed in ratios, proportions, and percents.

At all grade levels, students were assessed on their comprehension of number concepts and properties as well as their skills in addition, subtraction, multiplication, and division of whole numbers, simple fractions, and decimals. This included their knowledge of correct mathematical procedures and their ability to apply this knowledge to solve problems. At the eighth-grade level, students were required to demonstrate skill with whole numbers, fractions, decimals, integers, and rational numbers. Eighth-grade students also were assessed on their ability to use ratios and proportions and on their ability to read, use, and apply scientific notation to represent large and small numbers. At grade 12, questions within this content strand covered real and complex numbers as well as operators such as powers and roots. Students at all grades were assessed on their ability to reason mathematically and to communicate the reasoning they used to solve problems involving number sense, properties, and operations.

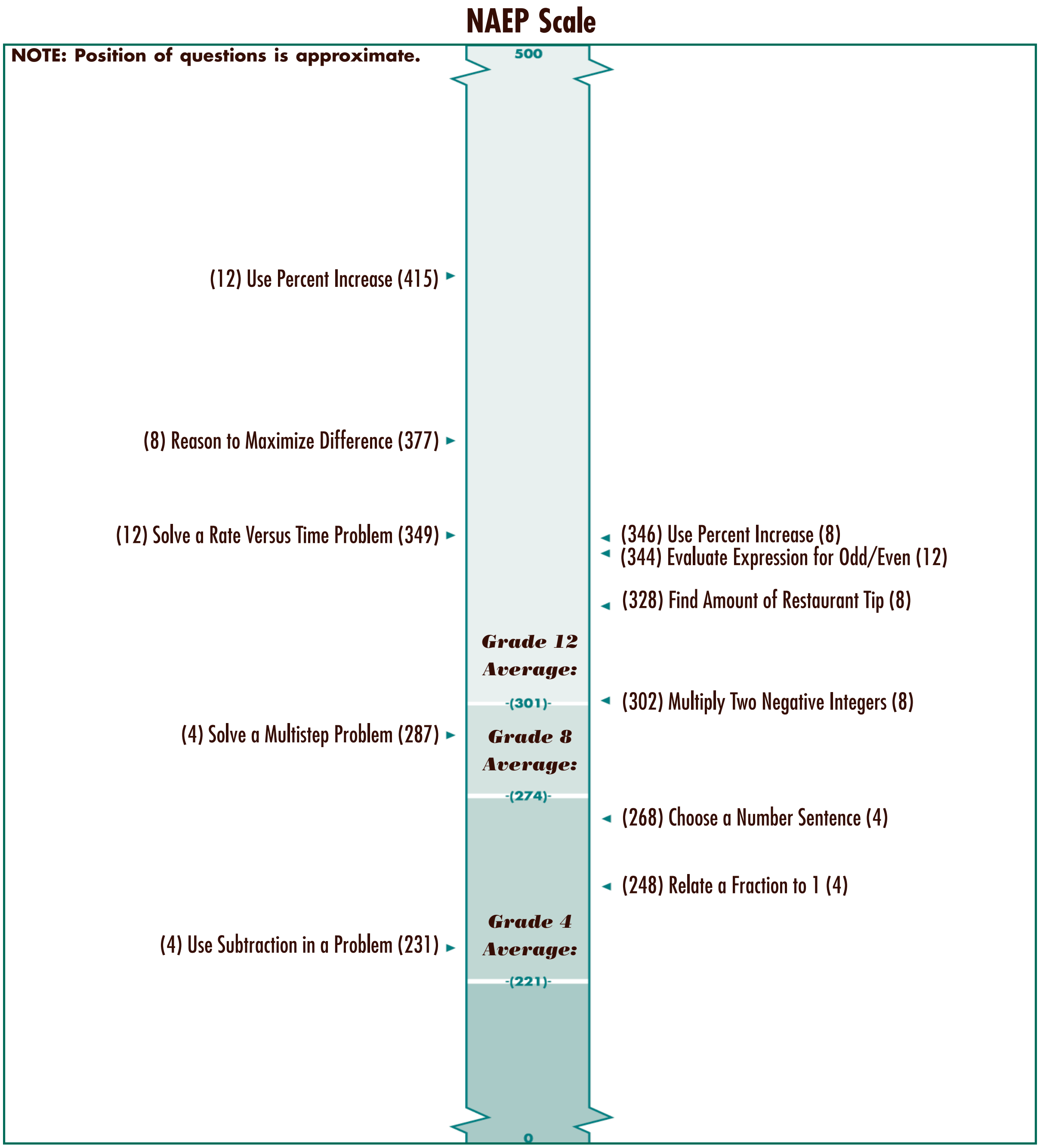
Examples of Individual Questions and Student Performance

Some of the Number Sense, Properties, and Operations questions from the NAEP 1996 mathematics assessment follow. Presentation of the questions is organized around five areas of emphasis within the content strand of Number Sense, Properties, and Operations. The first three areas of emphasis are directly related to the mathematical abilities from the NAEP mathematics framework. Thus, the area of *number meanings, properties, and other number concepts* includes questions that assessed a student's conceptual understanding of numbers and related number concepts; the area of *computation* includes questions that assessed a student's procedural knowledge of number operations; and the area of *application of computations* includes questions that assessed a student's problem-solving abilities. Two additional areas, *rounding and estimation* and *fractions, ratios, and proportions,* include questions that measured students' abilities to use skills specifically related to these two topics. Questions within all five areas also required students to reason, communicate, and make connections.

All sample questions from this content strand are mapped onto the NAEP composite mathematics scale as shown in Figure 3.1. Specific instructions on how to interpret this map are given at the end of Chapter 2. The map is included to provide a visual summary of the relative difficulty of each sample question and, thus, of the type of material mastered within this content strand by students with varying degrees of mathematics proficiency. Keep in mind, however, that the difficulty of a question is influenced by many factors, including characteristics specific to the question (e.g., format, absence or presence of graphics, real-world application) as well as the particular mathematics content associated with the question and student opportunities to learn this content. Remember also that overall performance on the Number Sense, Properties, and Operations content strand is not determined solely by performance on the examples presented here. These examples illustrate only some of what students know and can do.

Figure 3.1

Map of Selected Number Sense, Properties, and Operations Questions on the NAEP Composite Mathematics Scale (Item Map)



NOTE: Each mathematics question was mapped onto the NAEP 0 to 500 mathematics scale. The position of the question on the scale represents the scale score obtained by students who had a 65 percent probability of successfully answering the question. (The probability was 74 percent for a 4-option multiple-choice question and 72 percent for a 5-option multiple-choice question.) Only selected questions are presented. The number 4, 8, or 12 in parentheses is the grade level at which the question was asked.
NOTE: The map values for the question "Use Percent Increase" are very different for grades 8 and 12. This is because the question was treated differently in the estimation of achievement at the two grades. See discussion in text.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Asssessment.

Number meanings, properties, and other number concepts

Questions in this area required students to demonstrate a conceptual understanding of numbers. A relatively small proportion of the questions in the content strand fell into this area.

The questions for fourth-grade students that fell into this area tapped concepts related to the relative size of numbers, place value of whole and decimal numbers, and basic multiples (e.g., of 5 and 10). The questions for eighth-grade students measured their understanding of odd and even numbers and the properties of these numbers. More difficult questions included concepts of scientific notation and power. Twelfth-grade questions asked students to perform manipulations of place value and to apply their understanding of numbers to mathematical problems.

The following example from the twelfth-grade assessment tested students' understanding of the attributes of odd and even numbers and required a short constructed response.

If x and y are integers, then the expression $4x + 5y$ has a value that is odd or even depending on the values of x and y. For example, if x and y are each even, $4x$ is even and $5y$ is even. Therefore, $4x + 5y$ is also even. Fill in each of the blank spaces in the following table with either "odd" or "even" for the value of $4x + 5y$.

Value of x	Value of y	Value of $4x + 5y$
even	even	even
even	odd	
odd	even	
odd	odd	

The correct answers in descending vertical order are "odd," "even," and "odd."

This question assessed student understanding of what happens when odd and even numbers are multiplied and when they are summed. Students could have answered the question by trial and error (i.e., inserting various combinations of odd and even numbers into the equation). However, students could have responded more quickly if they had realized that odd numbers result only by multiplying two odd numbers or adding an even and an odd number. The question also incorporated symbolic (algebraic) notation and, thus, also evaluated a student's ability to make connections across content strands. If all three entries in the table were correctly listed as "odd," "even," "odd," the response was considered "correct." All other responses were considered "incorrect."

Student performance is reported in Tables 3.1 and 3.2. The title of the question (in quotation marks) can be used to locate the question on the item map in Figure 3.1. Thirty-eight percent of the students correctly responded to the question; that is, they had three correct entries in their tables. Fifty-seven percent had "incorrect" responses to the question, with 54 percent having one or two correct entries in their tables and 3 percent having no correct entries.[1] Six percent of students did not attempt the question. As might be expected, students who had taken more advanced mathematics courses were more likely to respond correctly to the question than students who had taken less advanced courses. For example, among twelfth-grade students who had taken or were taking calculus, 68 percent responded correctly to this question as opposed to only 16 percent of the students who had taken no algebra courses beyond pre-algebra.

Table 3.1 **Score Percentages for "Evaluate Expression for Odd/Even"**

THE NATION'S REPORT CARD naep

	Correct	Incorrect		Omit
	3 Correct Entries	1 or 2 Correct Entries	No Correct Entries	
Grade 12				
Overall	38	54	3	6
Males	36	54	4	7
Females	39	54	2	5
White	40	54	2	5
Black	36	51	6	7
Hispanic	22	63	5	10
Asian/Pacific Islander	57	35	2	6
American Indian	***	***	***	***
Geometry Taken	41	51	2	5
Highest Algebra-Calculus Course Taken:				
Pre-Algebra	16	70	3	9
First-Year Algebra	26	62	5	6
Second-Year Algebra	41	53	2	4
Third-Year Algebra/Pre-Calculus	53	41	2	5
Calculus	68	23	1	8

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

[1] Student responses for this and all other constructed-response questions also could have been scored as "off task," which means that the student provided a response, but it was deemed not related in content to the question asked. There are many examples of these types of responses, but a simple one would be "I don't like this test." Responses of this sort could not be rated. In contrast, responses scored as "incorrect" were valid attempts to answer the question that were simply wrong.

When performance is disaggregated by achievement level, Table 3.2 shows that 17 percent of students below the *Basic* level, 40 percent of students at the *Basic* level, and 67 percent of those at the *Proficient* level filled in their tables correctly. The question mapped at score 344 on the NAEP composite mathematics scale, meaning that students who scored 344 or above on the overall NAEP scale could likely fill in the table correctly.

Table 3.2 **Percentage Correct Within Achievement-Level Intervals for "Evaluate Expression for Odd/Even"**

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
38	17	40	67	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Computational skills

Questions that fell within the area of computational skills assessed students' procedural knowledge of number operations. These questions ranged from those that presented students with a number sentence (e.g., 2 + 5 = __) and required them to solve for the missing number, to more complex questions that might have required one, two, or multiple steps (i.e., operations). In some cases, students might have needed to recognize the order in which the steps were to be completed. However, in all cases, the operations to be performed were made explicit for the student. Computation questions were inherently context free; that is, they were not tied to any real-life situation or problem. They required the student to perform more or less routine calculations.

Computation questions at fourth grade primarily tended to be one or two steps and required the student to add, subtract, multiply, or divide whole or, sometimes, decimal numbers. At times, students were asked to perform two operations, but the order in which the operations were performed typically did not matter. At eighth grade, the calculations began to include negative numbers and the use of parentheses to designate the order in which operations needed to be performed. Some twelfth-grade questions included the use of exponents or algebraic notation and typically involved larger numbers than did questions at lower grade levels.

The following example is an eighth-grade question. The question required the student to multiply two negative numbers. It was a multiple-choice question and tested procedural knowledge of multiplication. Additionally, in order to respond correctly to the question, students needed to understand that the use of parentheses in this question indicated multiplication, to recognize that the computation involved negative numbers, and to know that the product of two negative numbers is a positive number. The question mapped at a score of 302 on the NAEP composite mathematics scale.

3. $(-5)(-7) =$

(A) -35
(B) -12
(C) -2
(D) 12
(E) 35

The correct option is E.

Performance data for this question are presented in Tables 3.3 and 3.4. Fifty percent of the students selected the correct option. Twenty percent of the students chose Option A, suggesting correct multiplication of the absolute value of the numbers but lack of knowledge of how to multiply negative numbers. Another 14 percent chose Option B, suggesting a lack of understanding of the arithmetic operation they were to perform.

Familiarity with negative numbers may depend on the student's curriculum. When student performance was examined by course enrollment, students in eighth-grade mathematics had the most difficulty with the question. Students in pre-algebra performed better than those in eighth-grade mathematics, and those in algebra performed better than students in both eighth-grade mathematics and pre-algebra.

Table 3.3 ***Percentage Correct for "Multiply Two Negative Integers"***

THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
Overall	50
Males	47
Females	54
White	55
Black	41
Hispanic	34
Asian/Pacific Islander	– –
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	35
Pre-Algebra	51
Algebra	75

*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

As shown in Table 3.4, 56 percent of those at the *Basic* level, 79 percent of those at the *Proficient* level, and 94 percent of those at the *Advanced* level selected the correct option. Only 25 percent of the students functioning below the *Basic* level responded correctly to this question.

Table 3.4 **Percentage Correct Within Achievement-Level Intervals for "Multiply Two Negative Integers"**

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 8 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
50	25	56	79	94

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Application of computational skills

Questions in this area assessed students' abilities to apply their computational skills to solve real-life problems. These questions were of the type traditionally referred to as word or story problems. What distinguished these questions from basic computation questions is that the questions were placed in a real-life context, requiring students to determine what operations they needed to perform and what numbers they needed to use in those operations. Sometimes the questions also presented extraneous or irrelevant information. Most often, students needed to identify and perform an arithmetic operation to arrive at an answer to the problem presented; however, at times they were simply asked to identify a number sentence that would lead to a correct solution. Again, arriving at a solution could entail one or several steps. Fourth-grade questions could often be solved in one or two steps, required simple computations, and presented little extraneous information. At grades 8 and 12, the questions involved more complex computations, required several steps, and presented more information for the student to synthesize. A fairly large proportion of the questions at all three grades fell into this area of emphasis, although they may have required the student to use skills in other areas such as rounding or proportional reasoning as well.

Three examples are presented for this area. The first two examples are fourth-grade multiple-choice questions, and the third is an eighth-grade extended constructed-response question.

The first question provided students with information about a partially completed driving trip and asked them to determine the remaining distance to be driven. In order to compute the number of miles left, students had to identify which numbers were extraneous and which were essential to the calculation, recognize that they needed to subtract, and know which number to subtract from the other; they then had to perform the subtraction correctly. Thus, the question also assessed mathematical reasoning and procedural knowledge in addition to problem-solving ability. It was a fairly easy question and mapped at a composite scale score of 231.

1. Kitty is taking a trip on which she plans to drive 300 miles each day. Her trip is 1,723 miles long. She has driven 849 miles. How much farther must she drive?

(A) 574 miles
(B) 874 miles
(C) 1,423 miles
(D) 2,872 miles

Did you use the calculator on this question?

○ Yes ○ No

The correct option is B.

Student performance data are presented in Table 3.5, and the percentage of students within each achievement-level interval who successfully answered the question is presented in Table 3.6. Sixty-four percent of the students answered the question correctly. Incorrect responses were evenly distributed across the other options. Seventy-five percent of students at the *Basic* level and more than 90 percent of students at the *Proficient* level selected the correct response.

Table 3.5 ***Percentage Correct for "Use Subtraction in a Problem"***

THE NATION'S REPORT CARD naep

Grade 4	Percentage Correct
Overall	64
Males	65
Females	63
White	71
Black	43
Hispanic	53
Asian/Pacific Islander	***
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 3.6 ***Percentage Correct Within Achievement-Level Intervals for "Use Subtraction in a Problem"***

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 4 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
64	34	75	94	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The second question also is a multiple-choice question for fourth-grade students. It presented the student with a graphic of flowerpots arranged in five rows and four columns. The student needed to determine which of the four number sentences would enable "Kevin" to calculate the number of seeds needed if three seeds were to be placed in each pot. The question also assessed the student's understanding of operations in that the student needed to know that a correct answer required Kevin to multiply the number of seeds by the number of rows and the number of pots per row. The question mapped at a score of 268 on the composite mathematics scale.

5. The picture shows the flowerpots in which Kevin will plant flower seeds. He needs 3 seeds for each pot. Which of the following number sentences shows how many seeds Kevin will need for all of the pots?

Ⓐ $5 \times 4 \times 3 = \square$

Ⓑ $(5 \times 4) + 3 = \square$

Ⓒ $(5 + 4) \times 3 = \square$

Ⓓ $5 + 4 + 3 = \square$

The correct option is A.

Student data for this question are presented in Tables 3.7 and 3.8. Fifty percent of the students answered the question correctly; however, 25 percent of the students chose Option B as the correct response. These students may have recognized that they needed to multiply the number of rows by columns in order to determine the number of flowerpots, but failed to recognize they also needed to multiply by the number of seeds.

Table 3.7

Percentage Correct for "Choose a Number Sentence"

THE NATION'S REPORT CARD naep

Grade 4	Percentage Correct
Overall	50
Males	50
Females	50
White	53
Black	42
Hispanic	45
Asian/Pacific Islander	45
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

According to Table 3.8, a correct response to this question was provided by approximately three-quarters of the fourth-grade students classified as *Proficient*, half of those classified as *Basic*, and 30 percent of those classified as below *Basic*.

Table 3.8

Percentage Correct Within Achievement-Level Intervals for "Choose a Number Sentence"

THE NATION'S REPORT CARD naep

	NAEP Grade 4 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
50	30	53	74	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

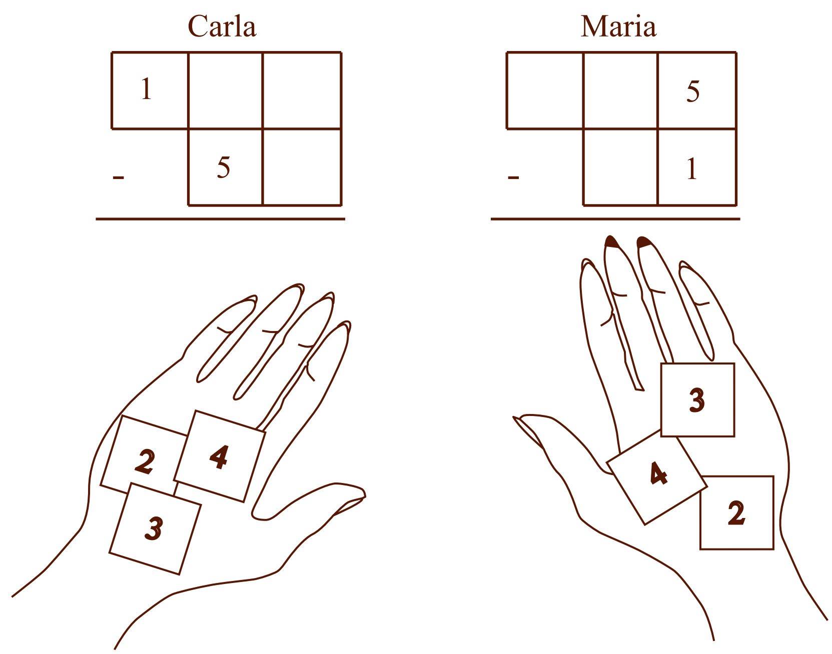
The next example is a problem-solving question for eighth-grade students that was set in the context of a mathematical game. Students in today's classrooms often are presented with such games, sometimes referred to as brain twisters, mind benders, or math challenges. This question involved an extended constructed response, requiring the student not only to reason, but also to communicate mathematically.

The question first presented students with some general directions explaining that it was important for them to show their work and explain their reasoning so that someone reading their response could understand their thinking. Next, students were shown pictures representing the ways two players had placed their numbered tiles for a subtraction problem and were told that the player with the largest answer would win the game. Students then were asked to state who would win the game and to explain how they knew that person would win.

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show <u>all</u> your work.

13. In a game, Carla and Maria are making subtraction problems using tiles numbered 1 to 5. The player whose subtraction problem gives the largest answer wins the game.

Look at where each girl placed two of her tiles.



Who will win the game? ______________________________

Explain how you know this person will win.

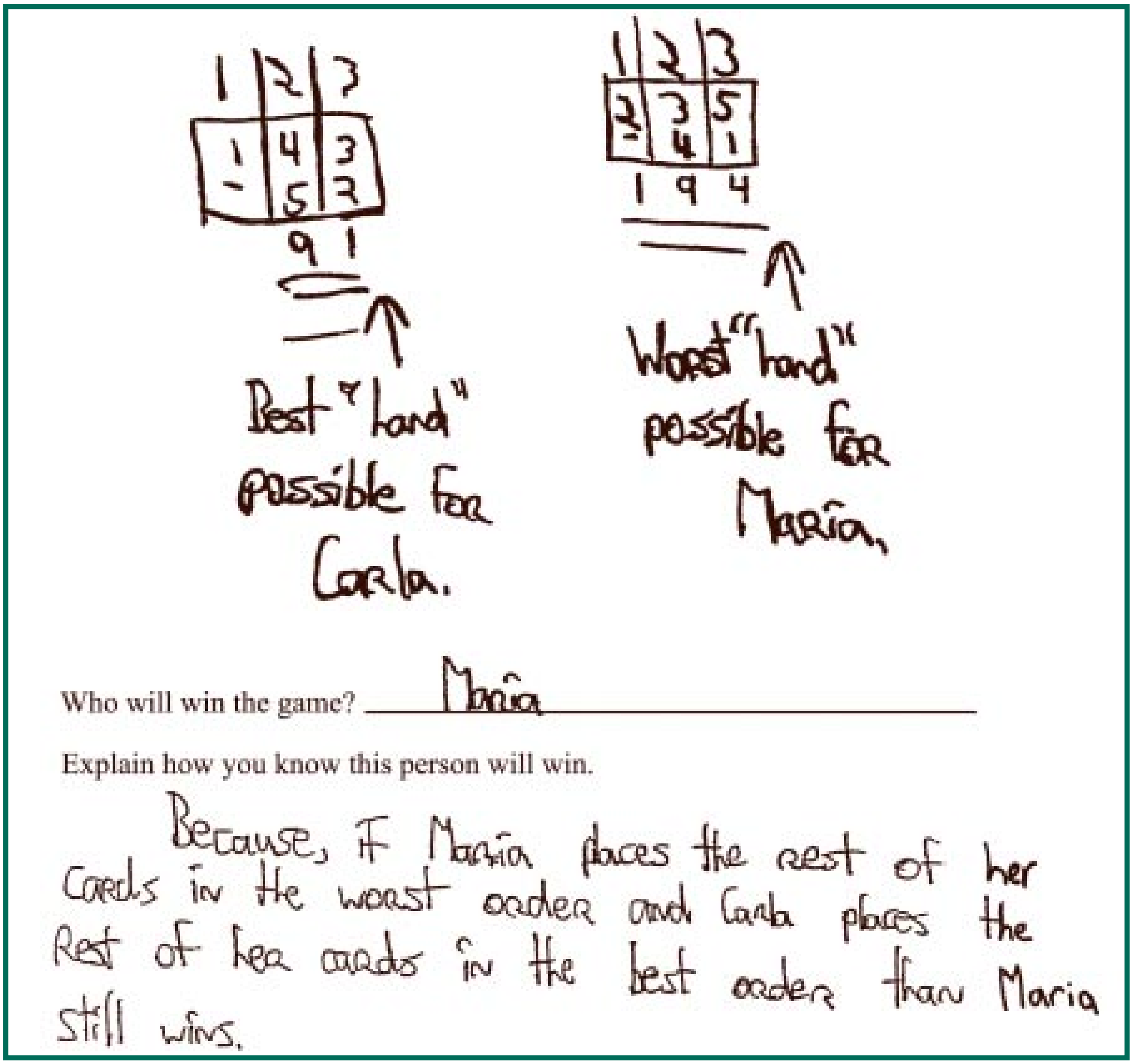
The correct answer is Maria.

In rating student responses, readers could rate a response as “extended,” “satisfactory,” “partial,” “minimal,” or “incorrect.” Students whose responses were considered to be “extended” correctly identified Maria as the winner by describing the answers to the subtraction problems. Examples of “extended” explanations included:

- The largest possible difference for Carla is less than 100, and the smallest possible difference for Maria is 194.
- Carla will only get a difference of 91 or less, but Maria will get several larger differences.

A sample “extended” response follows. This student displayed Carla’s best possible hand and Maria’s worst possible hand, labeled each as such, and explained that if these hands were played, Maria would win. This student clearly demonstrated to a reader the reasoning required to reach the correct conclusion.

Sample “extended” response

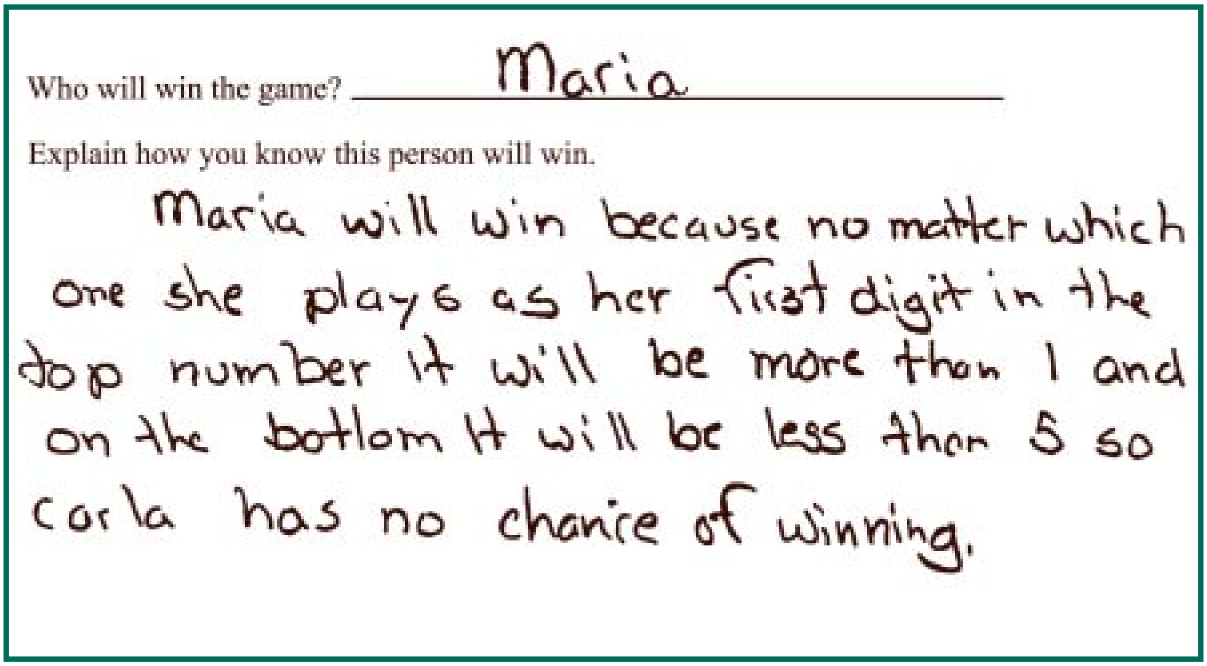


A response was considered to be “satisfactory” if the student correctly identified Maria as the winner and gave an explanation that indicated the rudimentary elements of a correct generalization. Acceptable “satisfactory” explanations included:

- Carla can have only up to 143 as her top number, but Maria can have 435 as her largest number.
- Carla has only one hundred, but Maria can have two, three, or four hundreds.
- Maria can never take away as much as Carla.

In the sample “satisfactory” response that follows, the student recognized and stated that Maria would always win because her top number would always be higher than Carla’s and her bottom number always lower.

Sample “satisfactory” response


Who will win the game? Maria

Explain how you know this person will win.

Maria will win because no matter which one she plays as her first digit in the top number it will be more than 1 and on the bottom it will be less than 5 so Carla has no chance of winning.

A response was considered “partial” if the student correctly identified Maria as the winner of the game but provided a partially correct or incomplete explanation. For example, in the following sample response, the student explained that Carla “made a mistake” by putting the “1” in the hundreds place, but did not complete the explanation by telling why this was a mistake and, thus, did not explain why Maria would always win.

Sample “partial” response

Who will win the game? Maria

Explain how you know this person will win.

because I to on maria's side 431 on Canla's side she

$$\begin{array}{r} 431 \\ -\ 21 \\ \hline 414 \end{array}$$

already made a Mistake by putting the 1 in first at the top. So then I took 143 on Canla's

$$\begin{array}{r} 143 \\ -\ 52 \\ \hline 91 \end{array}$$

side, wich leaves Maria as the winner

A response was considered “minimal” if it correctly identified Maria as the winner of the game but included no explanation, an incorrect explanation, or some response that did not enable the reader to determine how the student reached the conclusion. The following “minimal” response provides an example of a student who correctly identified the winner of the game but failed to explain why Maria’s score could never be lower than Carla’s. Thus, a reader would be unable to determine if the student arrived at the conclusion simply by randomly placing numbers in the squares or whether the student truly understood why Maria had to win.

Sample “minimal” response

Who will win the game? M'IRA

Explain how you know this person will win.

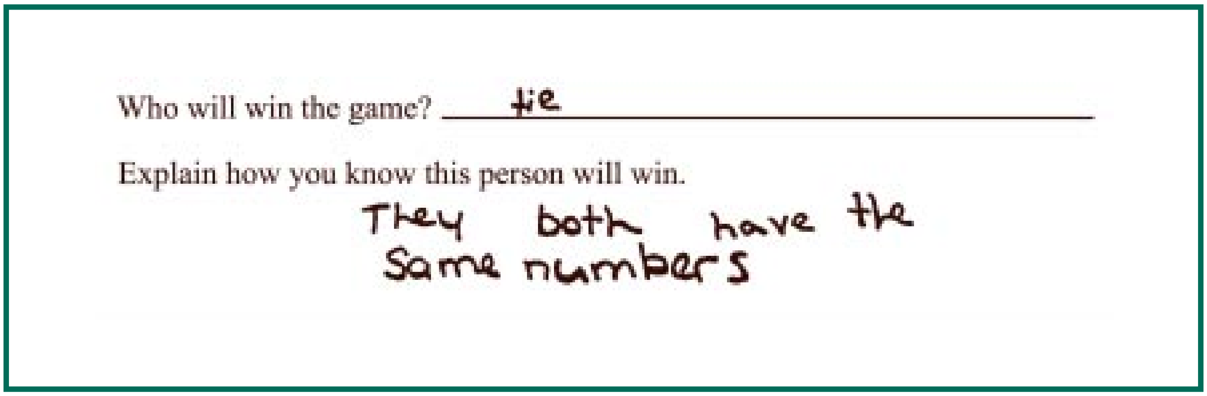
Becaus she has

$$\begin{array}{r} 435 \\ -\ 21 \\ \hline 414 \end{array}$$

hers is hier then Carla

To be evaluated as anything other than "incorrect," students' responses had to correctly identify Maria as the winner of the game. That is, "incorrect" answers were answers that indicated an outcome other than Maria winning the game. The following response is an example of an "incorrect" response.

Sample "incorrect" response


Who will win the game? tie

Explain how you know this person will win.

They both have the same numbers

Information on student performance in this question is presented in Tables 3.9 and 3.10. This question was quite difficult for students, and when the question was anchored to the NAEP scale, the "extended" and "satisfactory" rating categories were collapsed. While most eighth graders (95%) attempted to answer the question, only 15 percent provided a response that was rated at least "satisfactory." Another 16 percent provided responses rated "partial," and more than 60 percent provided responses rated "minimal" or "incorrect."

Table 3.9 — ***Score Percentages for "Reason to Maximize Difference"***

THE NATION'S REPORT CARD naep

Grade 8	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
Overall	1	14	16	32	31	5
Males	1	12	14	32	35	5
Females	1	17	19	32	26	4
White	1	17	18	29	29	5
Black	0!	7	10	41	36	5
Hispanic	0!	4	17	35	37	5
Asian/Pacific Islander	– –	– –	– –	– –	– –	– –
American Indian	***	***	***	***	***	***
Mathematics Course Taking:						
Eighth-Grade Mathematics	1	9	14	32	37	6
Pre-Algebra	0	12	19	31	32	5
Algebra	2	24	19	32	20	3

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Only 13 percent of students at the *Basic* level, 29 percent of students at the *Proficient* level, and 61 percent of students at the *Advanced* level submitted a response that was considered at least "satisfactory." The question mapped at 377 on the NAEP composite mathematics scale.

Table 3.10

Percentage at Least Satisfactory Within Achievement-Level Intervals for "Reason to Maximize Difference"

	NAEP Grade 8 Composite Scale Range			
Overall	**Below Basic**	**Basic**	**Proficient**	**Advanced**
15	4	13	29	61

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Rounding and estimation

Some questions in the Number Sense, Properties, and Operations content strand assessed students' abilities to round numbers and to estimate. Questions of this nature were either presented abstractly as a number or set of numbers for the student to round, or were presented within the context of a real-life type of problem. Students at each grade were asked to round whole as well as decimal numbers. Questions asking students to apply their rounding and estimation skills often involved money.

The example is a fourth-grade short constructed-response question presenting the student with prices for lunch items and asking the student to indicate the minimum number of one-dollar bills needed to pay for lunch for a week if the same items were purchased every day.

7. Sam can purchase his lunch at school. Each day he wants to have juice that costs 50¢, a sandwich that costs 90¢, and fruit that costs 35¢. His mother has only $1.00 bills. What is the least number of $1.00 bills that his mother should give him so he will have enough money to buy lunch for 5 days?

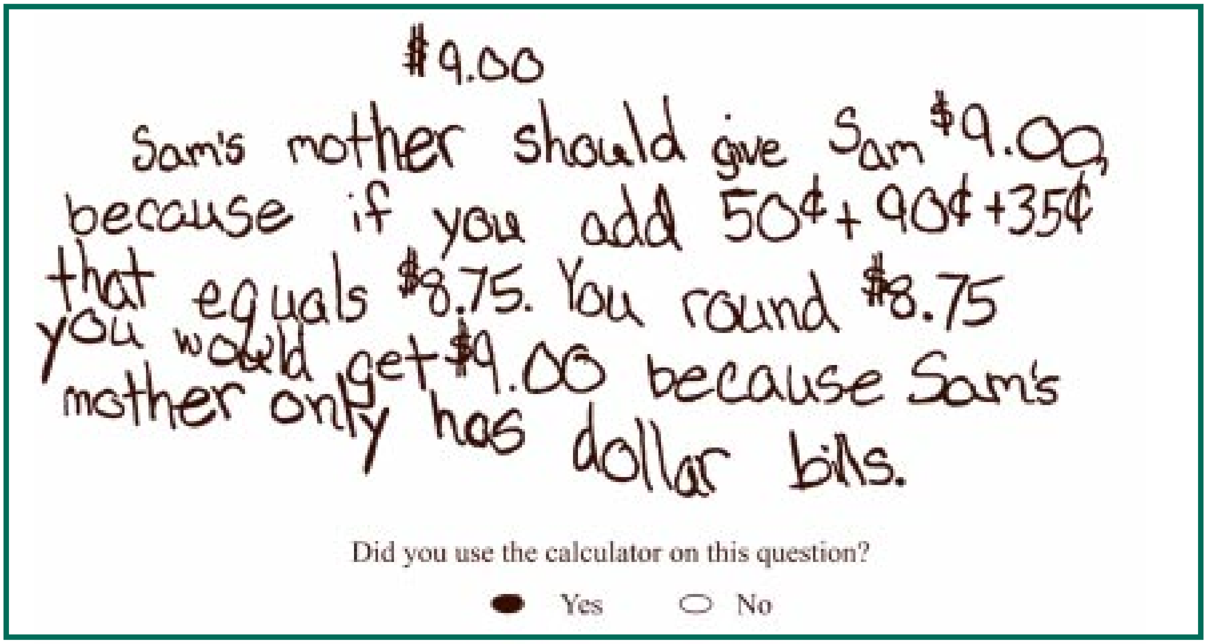
Did you use the calculator on this question?

◯ Yes ◯ No

The correct response is 9.

Responses were rated as "correct," "partial," or "incorrect." In order for a response to be rated as "correct," a student needed to add the cost of the items for a single day ($1.75) and then multiply this cost by 5 to determine the cost for 5 days ($8.75). Finally, the student needed to round this number to $9.00 and recognize that 9 one-dollar bills would be needed to buy lunch for a week, as shown in the following sample "correct" response. Note that the students were permitted to use calculators and were not required to show their work or provide an explanation in order for a response to be considered "correct." Simply writing down the number "9" would have been considered "correct."

Sample "correct" response


$9.00

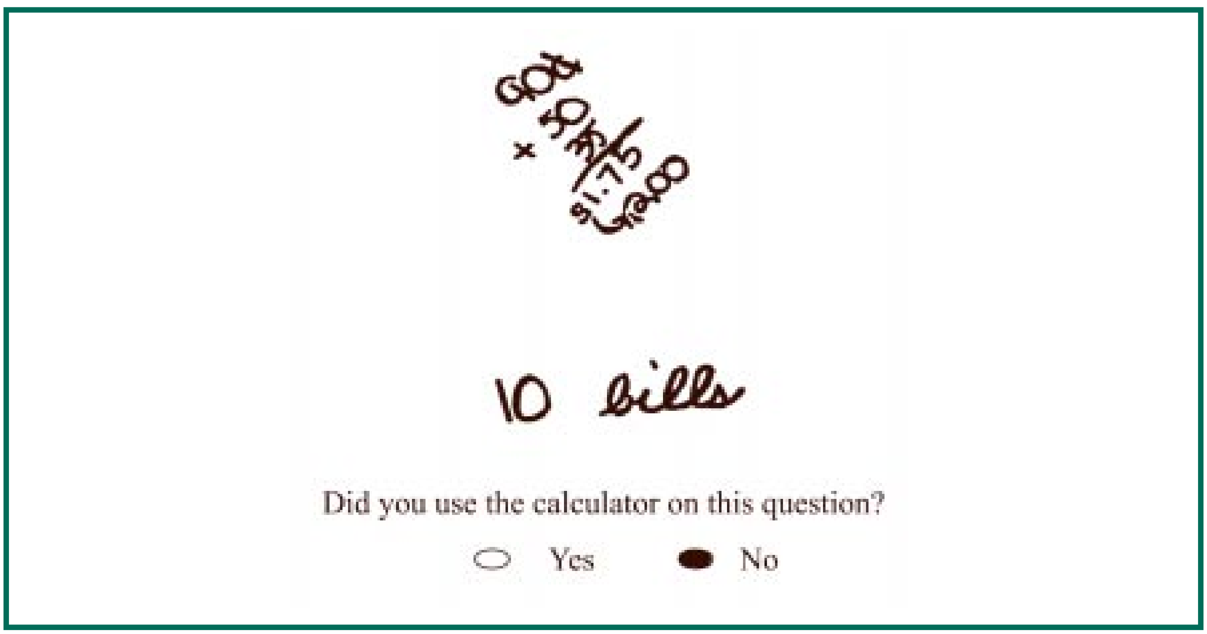
Sam's mother should give Sam $9.00, because if you add 50¢ + 90¢ + 35¢ that equals $8.75. You round $8.75 you would get $9.00 because Sam's mother only has dollar bills.

Did you use the calculator on this question?

● Yes ○ No

Rounding the weekly total down to $8.00 or estimating $2.00 each day for a total of $10.00 resulted in a response rated as "partial," as did small errors in computation. In the following sample response, the student correctly calculated the cost of lunch per day, but indicated rounding this number to $2.00. The student's final answer of "10 bills" was rated "partial."

Sample "partial" response


90¢

×

$1.75

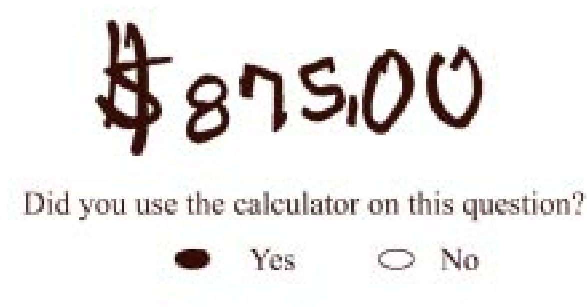
10 bills

Did you use the calculator on this question?

○ Yes ● No

All other answers were considered to be "incorrect." In the next sample response, the student presumably calculated the cost per day and for the week on the calculator. The student reached the correct total of "875" but did not place the decimal correctly.

Sample "incorrect" response


$875,00

Did you use the calculator on this question?

● Yes ○ No

Student data are presented in Tables 3.11 and 3.12. This question was difficult for most students. Ten percent of the students did not respond to the question, and half of the students responded incorrectly. The remaining students' responses split almost evenly between "correct" and "partial." Omitting the question was more common among Black students than among students from other racial/ethnic groups.

Table 3.11 ***Score Percentages for "Solve a Multistep Problem"***

Grade 4	Correct	Partial	Incorrect	Omit
Overall	17	20	51	10
Males	19	22	45	11
Females	15	18	57	8
White	21	23	47	7
Black	6	9	63	20
Hispanic	6	15	63	11
Asian/Pacific Islander	***	***	***	***
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Only 1 percent of grade 4 students classified as below *Basic* and 14 percent of those classified as *Basic* responded correctly to the question. Forty-four percent of those classified as *Proficient* responded correctly. The question mapped at 287.

Table 3.12 ***Percentage Correct Within Achievement-Level Intervals for "Solve a Multistep Problem"***

THE NATION'S REPORT CARD naep

	NAEP Grade 4 Composite Scale Range			
Overall	**Below Basic**	**Basic**	**Proficient**	**Advanced**
17	1	14	44	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Fractions, ratios, and proportions

The largest proportion of the Number Sense, Properties, and Operations questions measured student skills and knowledge in the areas of fractions, decimal fractions, percentages, ratios, and proportions. Many of these questions were among the most difficult for students. They included questions that required students to identify appropriate representations of common and decimal fractions, to order or identify equivalent fractions, and to apply their skills to computations involving fractions and percentages or problems involving proportional reasoning.

Fourth-grade questions covered representation, equivalence, and ordering of common fractions such as $^1/_2$ or $^1/_3$. Some of the more difficult questions involved decimals. Eighth-grade questions involved manipulation of more complex fractions, sometimes requiring the student to identify a least common denominator or to simplify the representation (i.e., reduce the fraction). Some questions required an understanding of the relationship between common and decimal fractions or involved the use of percentages. The twelfth-grade questions required students to exhibit such skills as explaining the relationship between common and decimal fractions and percentages, calculating fractions of fractions or interest, and reasoning with proportions in complex situations.

Four sample questions are presented for this area: one fourth-grade question, one eighth-grade question, one question that was presented at both eighth and twelfth grades, and one twelfth-grade question. The fourth-grade question was a short-answer question involving common fractions. The eighth-grade question involved calculation of a percentage. The eighth- and twelfth-grade question assessed student understanding of and ability to calculate percent increase. The twelfth-grade question was a rate versus time question. The example for grade 4 students follows.

1. How many fourths make a whole?
Answer:____________________

The correct answer is 4.

This question tested students' understanding of how fractions relate to a whole and required them to write a short response. The responses were rated "correct" or "incorrect," and a variety of responses such as "4," or "four fourths," or "4 fourths," etc., were accepted as "correct." Student performance data are presented in Table 3.13. Table 3.14 shows the percentage of students within each grade 4 achievement-level interval on the NAEP composite scale who successfully answered the question.

Table 3.13 ***Percentage Correct for "Relate a Fraction to 1"***

THE NATION'S REPORT CARD naep

Grade 4	Percentage Correct
Overall	50
Males	50
Females	50
White	57
Black	29
Hispanic	33
Asian/Pacific Islander	53
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 3.14

Percentage Correct Within Achievement-Level Intervals for "Relate a Fraction to 1"

	NAEP Grade 4 Composite Scale Range			
Overall	**Below Basic**	**Basic**	**Proficient**	**Advanced**
50	22	56	81	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Overall, 50 percent of fourth-grade students answered this question correctly. Sixteen percent of the students omitted the question. When results are presented by achievement level, 56 percent of students at the *Basic* level on the NAEP composite mathematics scale provided a correct response, whereas 81 percent at the *Proficient* level answered the question correctly. The question mapped at a scale score of 248 on the NAEP composite mathematics scale.

The second example is an eighth-grade question that asked students for the closest approximation of a 15 percent tip on a given restaurant bill. It required an understanding of both percent and estimation.

5. Of the following, which is the closest approximation of a 15 percent tip on a restaurant check of $24.99?

(A) $2.50
(B) $3.00
(C) $3.75
(D) $4.50
(E) $5.00

The correct option is C.

Student performance data for this question are presented in Table 3.15. This question was fairly difficult for eighth-grade students and mapped at a scale score of 328 on the NAEP composite mathematics scale. Only 38 percent of students chose the correct option, while approximately 20 percent of students chose Option A, and another 20 percent chose Option B. The performance suggests that students had difficulty calculating the requested percent, that they did not appreciate the level of precision required for a successful estimation, or that they simply responded with what they considered to be an appropriate tip without attending to the direction that the tip be 15 percent. Students currently taking pre-algebra or eighth-grade mathematics performed similarly, whereas those currently taking algebra performed better than students in the other two groups.

Table 3.15

Percentage Correct for "Find Amount of Restaurant Tip"

THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
Overall	38
Males	37
Females	39
White	38
Black	40
Hispanic	28
Asian/Pacific Islander	– –
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	34
Pre-Algebra	33
Algebra	48

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The percentage of students within each achievement-level interval who successfully answered the question is presented in Table 3.16. That the question was challenging for students can be seen by the fact that only 37 percent of eighth-grade students at the *Basic* level, 54 percent at the *Proficient* level, and 68 percent at the *Advanced* level on the NAEP composite mathematics scale answered the question correctly.

Table 3.16

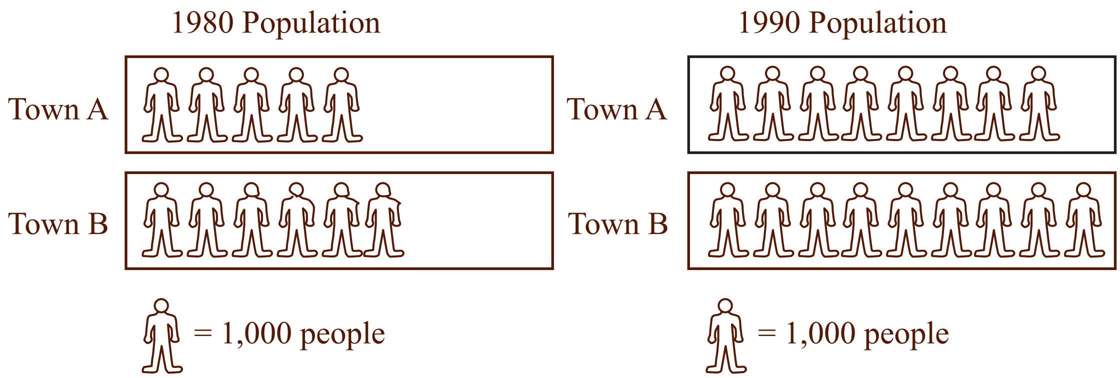
Percentage Correct Within Achievement-Level Intervals for "Find Amount of Restaurant Tip"

THE NATION'S REPORT CARD naep

	NAEP Grade 8 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
38	26	37	54	68

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The third example in this area is a problem-solving question that was administered to students in grades 8 and 12. It described the population growth of two towns, both textually and graphically, and gave two opinions (Brian's and Darlene's) regarding the relative growth of the two towns. Students were asked to use mathematics to explain how either opinion might be justified.



5. In 1980, the populations of Town A and Town B were 5,000 and 6,000, respectively. The 1990 populations of Town A and Town B were 8,000 and 9,000, respectively.

Brian claims that from 1980 to 1990 the populations of the two towns grew by the same amount. Use mathematics to explain how Brian might have justified his claim.

Darlene claims that from 1980 to 1990 the population of Town A had grown more. Use mathematics to explain how Darlene might have justified her claim.

Did you use the calculator on this question?

○ Yes ○ No

In the question, Brian offered a conclusion based on the fact that the absolute size of the population growth was the same for both towns. Acceptable mathematics for demonstrating Brian's conclusion included:

Town A	8,000 − 5,000 = 3,000		6,000 − 5,000 = 1,000
		or	
Town B	9,000 − 6,000 = 3,000		9,000 − 8,000 = 1,000

Darlene's conclusion was based on the proportional growth of the two towns, which was greater for Town A than Town B. Acceptable mathematics for demonstrating Darlene's conclusion included:

Town A $\quad \dfrac{8{,}000 - 5{,}000}{5{,}000} \times 100\% = 60\%$ $\quad$ or $\quad 8 \div 5 = 1.6$

Town B $\quad \dfrac{9{,}000 - 6{,}000}{6{,}000} \times 100\% = 50\%$ $\quad$ or $\quad 9 \div 6 = 1.5$

A response was rated as "correct" if the student provided a correct mathematical calculation (as illustrated above) for both Brian and Darlene. In the following "correct" example, the student provided correct mathematical explanations for both Brian's and Darlene's conclusions.

Sample "correct" response

BK in 1980 the populations of
A + B were 5 000 + 6000 respectively
and in 1990 the populations of
A+B were 8000 + 9000 respectively
so they both in creased by 3000 people
A 8000-5000=3000 + 9000-6000=3000

Town A increased by a greater percentage
than town B 3000=__% of 5000 =60%
3000=__% of 6000 =50%

Did you use the calculator on this question?

● Yes ○ No

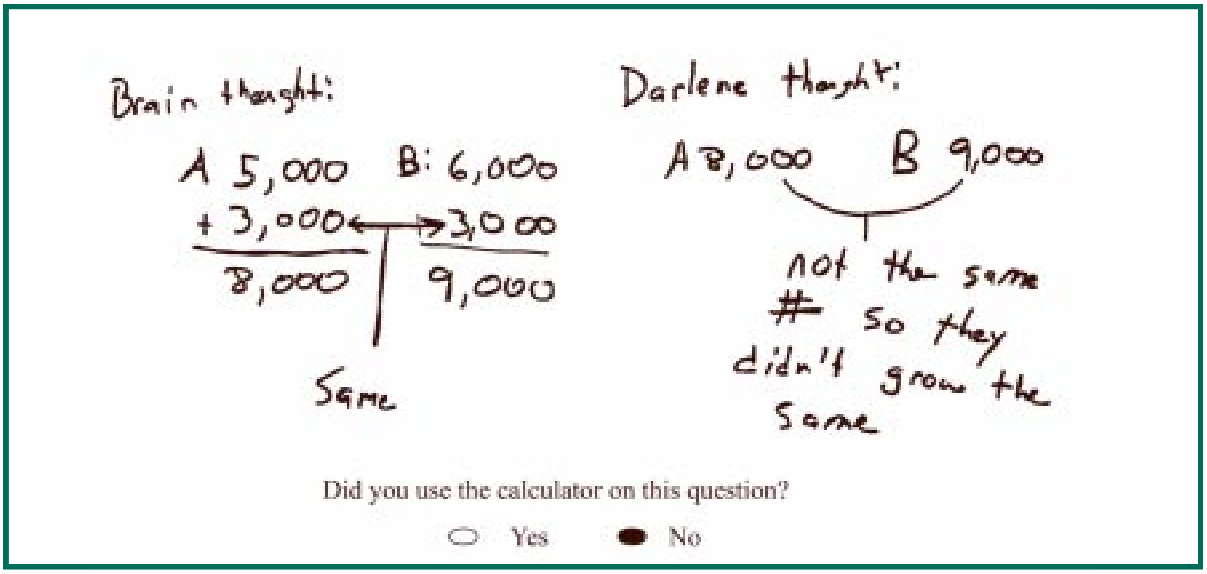
Student responses were rated as “partial” if they did one of the following:

- indicated Brian’s solution (either 1,000 or 3,000) and Darlene’s solution (60% and 50%) but did not show the mathematical explanation (calculation) that they used to arrive at these solutions; or

- indicated either Brian’s solution or Darlene’s solution with the correct mathematical explanation (calculation).

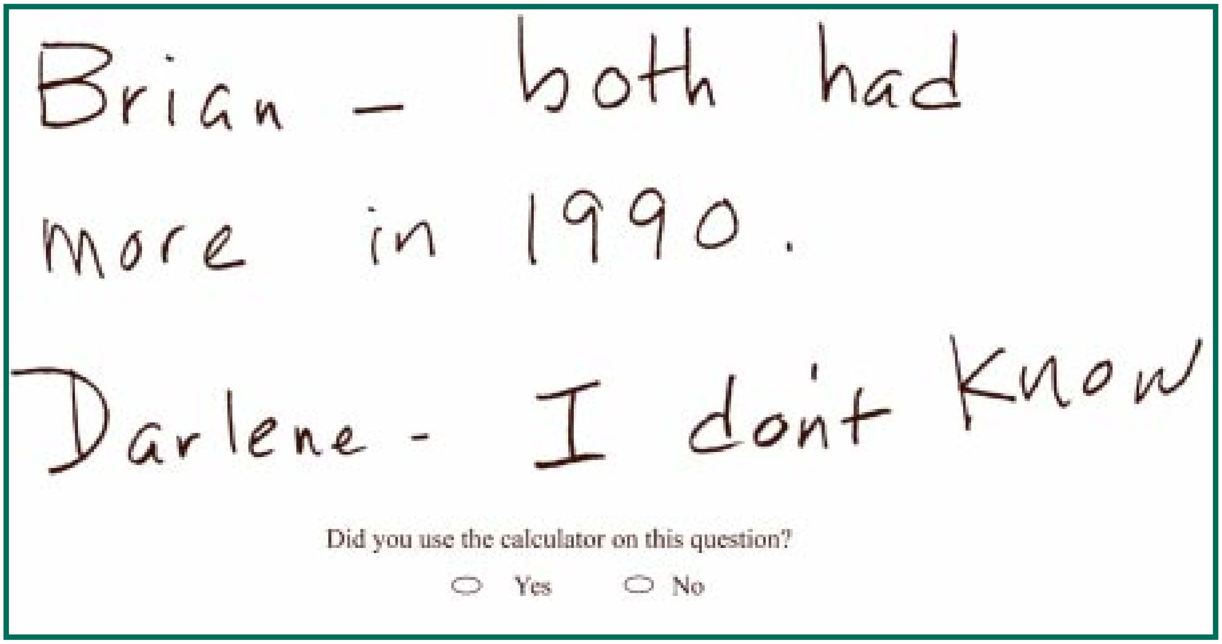
This next sample response was rated as a “partial” response. The student gave a variation of the 8,000 – 5,000 and 9,000 – 6,000 mathematical explanation presented above for Brian’s conclusion. However, the student did not provide a correct mathematical explanation for Darlene’s conclusion.

Sample “partial” response



"Incorrect" responses were those that were not considered to be at least partially correct. In the following sample "incorrect" response, the student provided no mathematical explanation for either Brian's or Darlene's conclusion.

Sample "incorrect" response



Brian – both had more in 1990.

Darlene - I don't know

Did you use the calculator on this question?

Yes No

Student performance data for both grades 8 and 12 are presented in Tables 3.17 and 3.18. Student performance on the question was similar across the two grades. One percent of eighth-grade students and 3 percent of twelfth-grade students provided responses that were rated "correct," and 21 percent of eighth-grade students and 24 percent of twelfth-grade students provided responses that were rated as "partial." Sixty percent of the responses at grade 8 and 56 percent of the responses at grade 12 were rated "incorrect."

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Table 3.17 — ***Score Percentages for "Use Percent Increase"***

	Correct	Partial	Incorrect	Omit
Grade 8				
Overall	1	21	60	16
Males	0	17	62	19
Females	1	26	58	13
White	1	24	62	11
Black	0!	14	57	28
Hispanic	0!	17	52	31
Asian/Pacific Islander	– –	– –	– –	– –
American Indian	***	***	***	***
Mathematics Course Taking:				
Eighth-Grade Mathematics	0!	15	66	16
Pre-Algebra	0!	21	58	18
Algebra	2	33	53	11
Grade 12				
Overall	3	24	56	16
Males	4	22	56	18
Females	2	27	56	14
White	4	25	60	11
Black	0!	21	50	26
Hispanic	2	18	46	34
Asian/Pacific Islander	5	45	31	17
American Indian	***	***	***	***
Geometry Taken	3	27	56	14
Highest Algebra-Calculus Course Taken:				
Pre-Algebra	***	***	***	***
First-Year Algebra	1	15	61	22
Second-Year Algebra	3	24	57	14
Third-Year Algebra/Pre-Calculus	4	39	53	4
Calculus	12	47	33	8

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Few students at any of the achievement levels for either grade provided "correct" responses to the question. The best performance was by twelfth-grade students at the *Proficient* level. Eleven percent of twelfth-grade students classified as *Proficient* provided "correct" responses.

For grade 12, the question mapped at 415. However, at grade 8, when the question was anchored to the NAEP scale, the "correct" and "partial" rating categories were collapsed. The collapsed response category mapped at 346 for grade 8 on the NAEP composite mathematics scale. In other words, whereas the highest response category ("correct") was mapped for grade 12, the lower collapsed category ("correct" plus "partial") was mapped for grade 8.

Table 3.18 ***Percentage Correct Within Achievement-Level Intervals for "Use Percent Increase"***

THE NATION'S REPORT CARD naep

		NAEP Grades 8 and 12 Composite Scale Ranges			
	Overall	Below Basic	Basic	Proficient	Advanced
Grade 8	1	0!	0!	2!	4!
Grade 12	3	0!	1	11	***

*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The final example is a twelfth-grade multiple-choice question. The question involved rate and time and tested students' knowledge of procedures used to solve for rate per unit of time.

3. A certain machine produces 300 nails per minute. At this rate, how long will it take the machine to produce enought nails to fill 5 boxes of nails if each box will contain 250 nails?

(A) 4 min
(B) 4 min 6 sec
(C) 4 min 10 sec
(D) 4 min 50 sec
(E) 5 min

The correct option is C.

This question was a multistep problem that a student could have solved in a number of ways. One possible approach was to determine how many nails were desired (5 boxes $\times$ 250 nails/box = 1,250 nails), then to solve for time required to produce 1,250 nails (1,250 nails/300 nails per minute). The solution is 4.16 minutes, which equals 4 minutes and 10 seconds. A proportional approach also could have been used. After determining the numbers of nails desired, a student could have solved the proportionality equation $300/60 = 1{,}250/x$ to get the time required.

Student performance data are presented in Tables 3.19 and 3.20. Almost half of the students answered the question correctly. Nineteen percent chose Option B, and 12 percent chose Option D. Male students performed better than females. This question mapped at 349 on the NAEP composite mathematics scale.

Table 3.19 ***Percentage Correct for "Solve a Rate Versus Time Problem"***

THE NATION'S REPORT CARD naep

Grade 12	Percentage Correct
Overall	49
Males	56
Females	43
White	53
Black	36
Hispanic	41
Asian/Pacific Islander	63
American Indian	***
Geometry Taken	52
Highest Algebra-Calculus Course Taken:	
Pre-Algebra	37
First-Year Algebra	49
Second-Year Algebra	48
Third-Year Algebra/Pre-Calculus	57
Calculus	65

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The question was answered correctly by 73 percent of the students classified as *Proficient*, 51 percent of the students classified as *Basic*, and 34 percent of the students classified as below *Basic*.

Table 3.20 **Percentage Correct Within Achievement-Level Intervals for "Solve a Rate Versus Time Problem"**

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
49	34	51	73	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Summary

Questions in this content strand assessed students' conceptual understanding of number meanings, properties, and other number concepts; procedural knowledge of number operations; and application of this understanding and knowledge to real-life problems. The understanding, knowledge, and application sometimes involved rounding, estimation, or proportional thinking. Questions assessing ratios and proportional thinking tended to be among the most difficult, and the computation questions tended to be among the easiest. Few questions required decontextualized computations. Rather, the questions often involved real-life situations presented either as a "story" or in graphics. Some questions asked students to round or estimate as one step in arriving at the solution.

The majority of students appeared to grasp many of the fundamental concepts of numbers, relationships between numbers, and properties of numbers, as well as to display the skills required for manipulating numbers and completing computations. Questions requiring multistep solutions or involving new concepts tended to be more difficult. Additionally, questions requiring students to solve problems and communicate their reasoning proved challenging, and often it was the communication aspect that provided the most challenge.